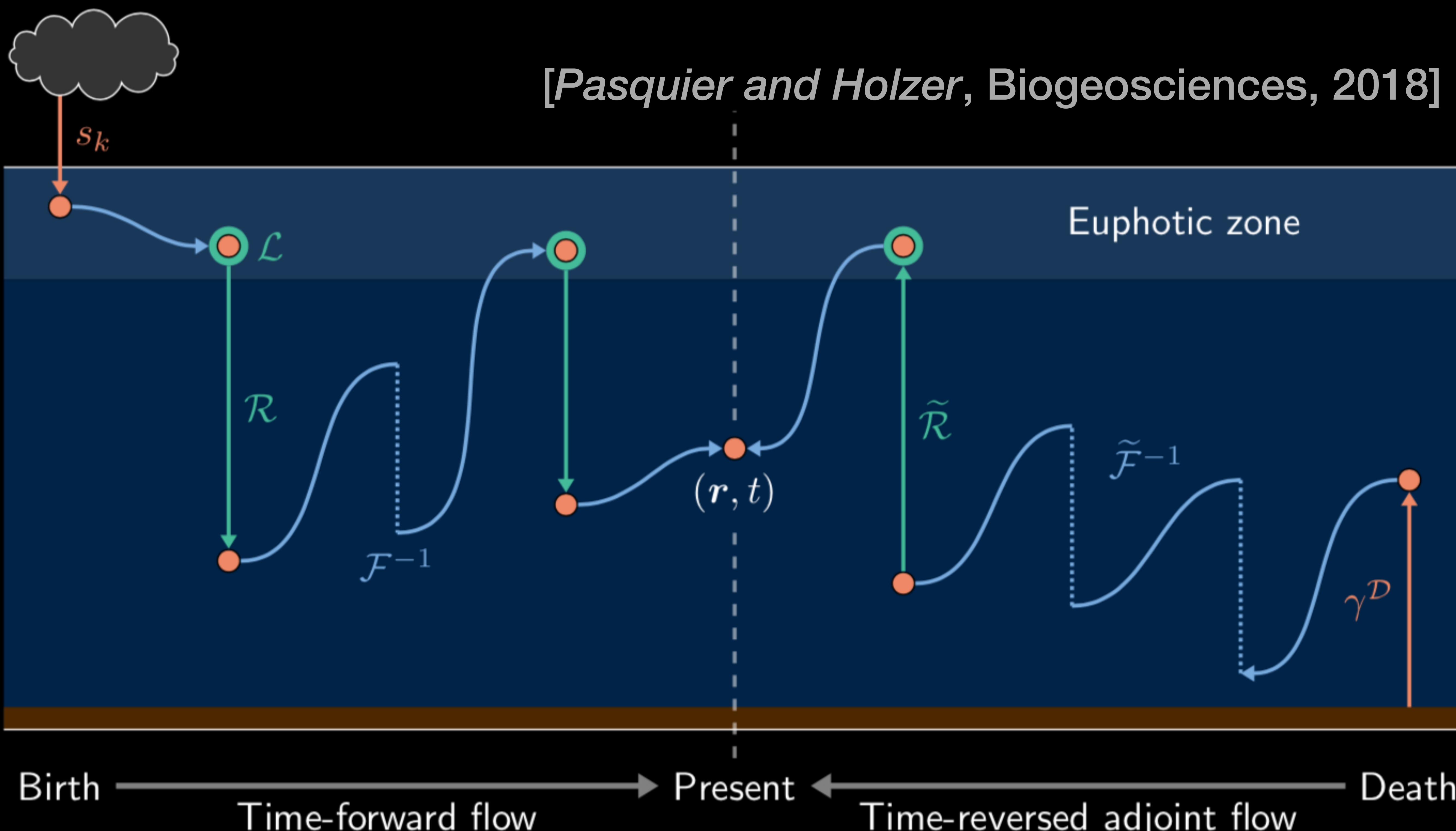
Friday 25 Jan 2019 Massachusetts Institute of Technology

The number of past and future regenerations of iron in the ocean and its intrinsic fertilization efficiency

[*Pasquier and Holzer*, Biogeosciences, 2018]



Questions

1. What fraction of the DFe distribution in the current state of the ocean has passed *n* times through the biological pump in the past, and what fraction will pass *m* times through the biological pump in the future, for any given *n* and *m*?

2. How much does DFe at any given location contribute to the global export production per DFe molecule?

3. How do the mean number of past and future passages through the biological pump, and the closely related iron fertilization efficiency, depend on the uncertain iron source strengths?

Tracer equation for DFe

DFe sources
(aeolian, sedimentary, hydrothermal)

$$(\partial_t + \mathcal{T})\chi = \sum_c (\mathcal{S}_c - 1) U_c(\chi) + \sum_j (\mathcal{S}_j - 1) J_j(\chi) + \sum_k s_k$$

Reversible scavenging

Uptake (biological pump feed)

Regeneration (biological pump exhaust)

Advection-diffusion

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$$(\partial_t + \mathcal{T})\chi = \mathcal{R}\chi - \mathcal{L}\chi - \mathcal{D}\chi + \sum_k s_k$$

Reversible scavenging

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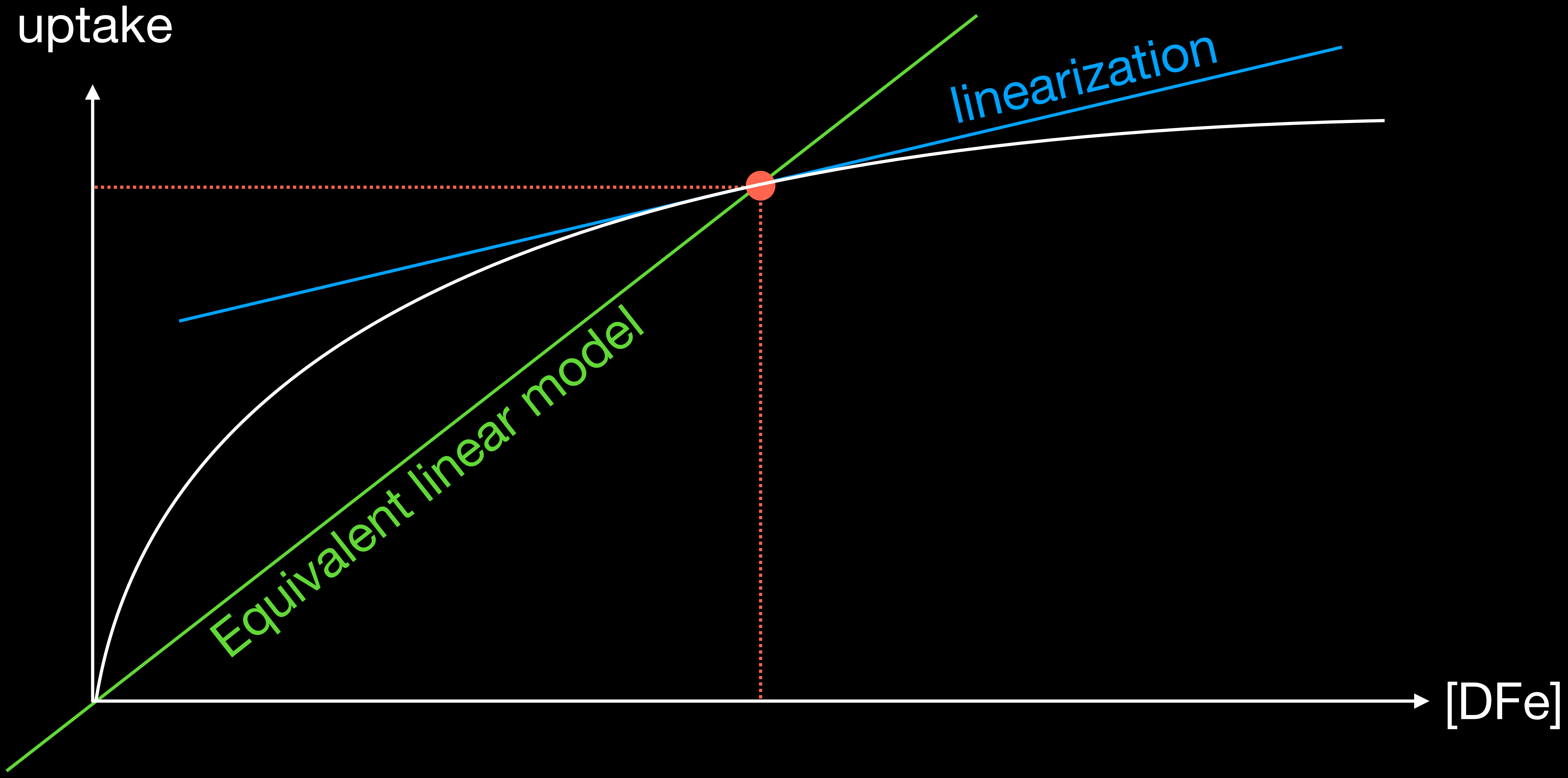
Regeneration (biological pump exhaust)

Advection-diffusion

The equivalent linear system is not a linearization of the nonlinear model!



[*Holzer et al*., 2016]

Partitioning DFe by source type

$$(\partial_t + \mathcal{T})\chi = \mathcal{R}\chi - \mathcal{L}\chi - \mathcal{D}\chi + \sum_k s_k$$

$$\mathcal{H} \equiv \mathcal{T} - \mathcal{R} + \mathcal{L} + \mathcal{D}$$

$$(\partial_t + \mathcal{H})\chi = \sum_k s_k$$

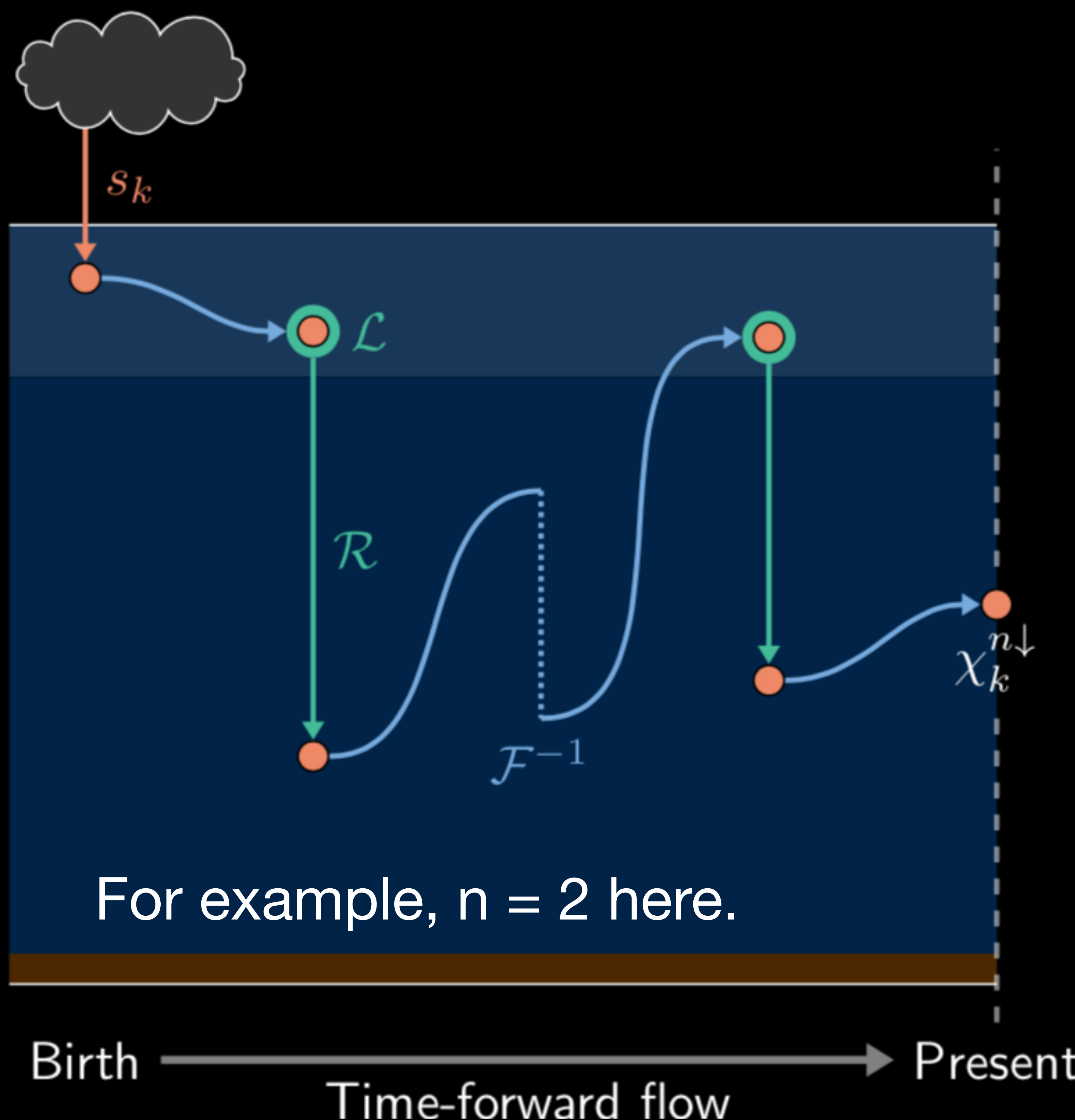
Steady state: $\mathcal{H}\chi = \sum_k s_k$

The DFe concentration, χ_k, that came from source s_k is given by $\mathcal{H}\chi_k = s_k$

In MATLAB (or Julia!) it is evaluated by `H \ sk`

Single matrix inversion!

Partitioning DFe by *n*, the number of past regenerations



Starting point of the recursion

$$(\partial_t + \mathcal{T})\chi = \cancel{\mathcal{R}\chi} - \mathcal{L}\chi - \mathcal{D}\chi + \sum_k s_k$$

$$(\partial_t + \mathcal{F})\chi^{0\downarrow} = \sum_k s_k \qquad \mathcal{F} \equiv \mathcal{T} + \mathcal{L} + \mathcal{D}$$

The operator $\mathcal{F}$ does not allow DFe to be regenerated. I.e., each DFe molecule that gets taken up is taken *out*:

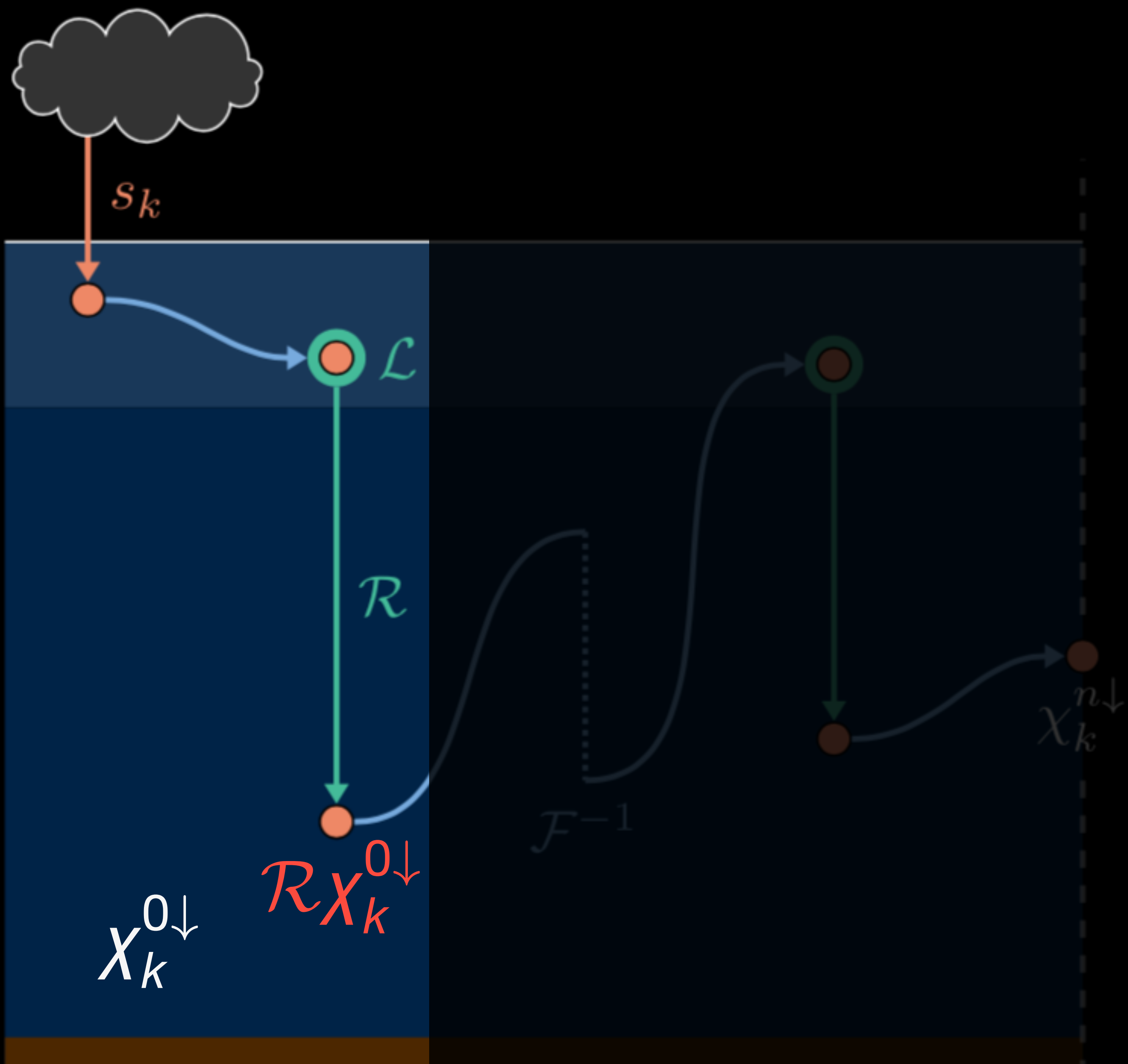
Thus, the steady state given by $\mathcal{F}\chi^{0\downarrow} = \sum_k s_k$

is exactly the DFe that was not regenerated in the past.

$$\mathcal{F}\chi_k^{0\downarrow} = s_k \qquad (1)$$

Recursion: $$\mathcal{F}\chi_k^{(n+1)\downarrow} = \mathcal{R}\chi_k^{n\downarrow} \qquad (2)$$

The rate of first regeneration is $\mathcal{R}\chi_k^{0\downarrow}$.

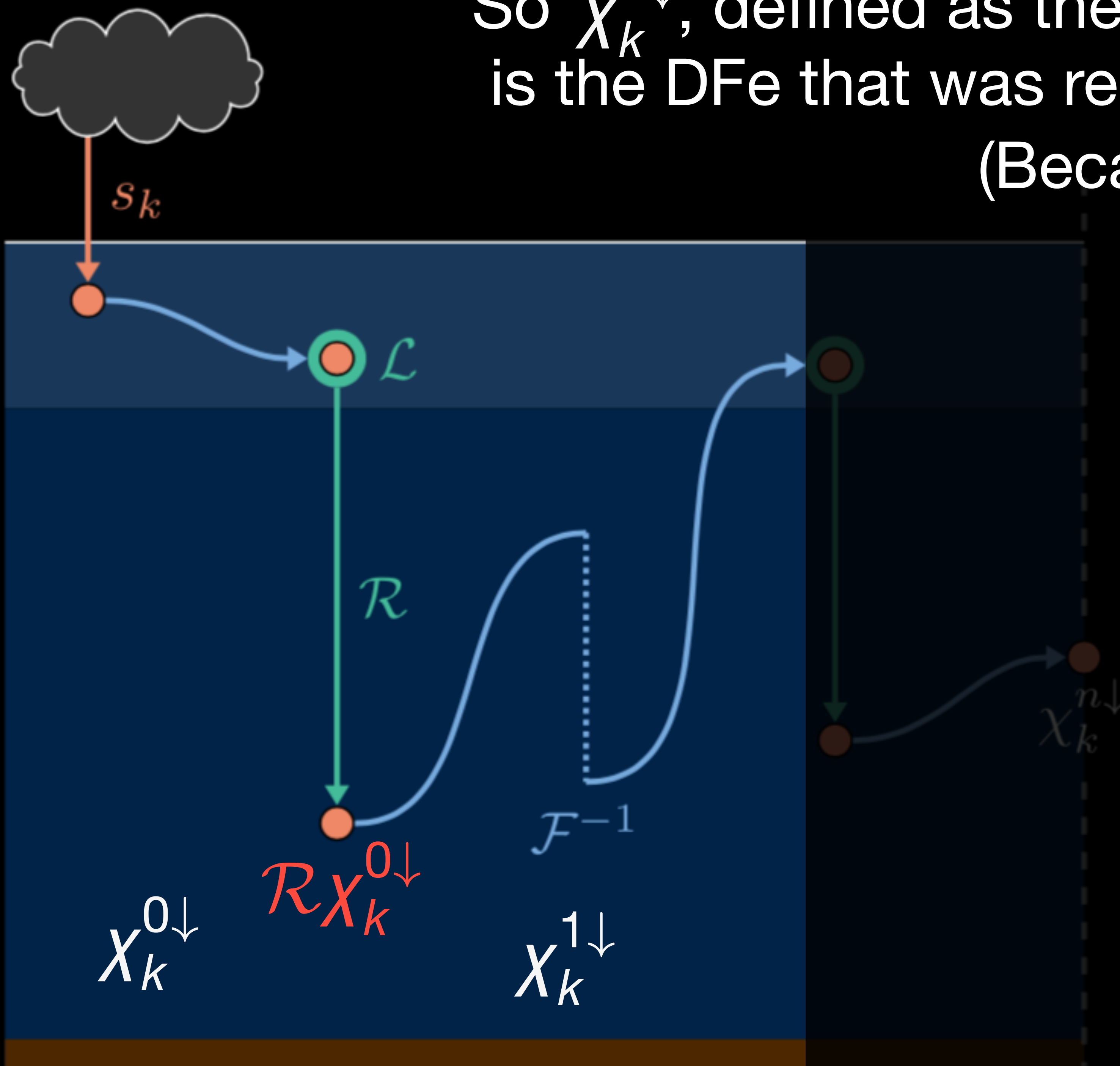


Recursion: $\mathcal{F}\chi_k^{(n+1)\downarrow} = \mathcal{R}\chi_k^{n\downarrow}$ (2)

The rate of first regeneration is $\mathcal{R}\chi_k^{0\downarrow}$.

So $\chi_k^{1\downarrow}$, defined as the solution to $\mathcal{F}\chi_k^{1\downarrow} = \mathcal{R}\chi_k^{0\downarrow}$, is the DFe that was regenerated exactly once.

(Because $\mathcal{F}$ = no regeneration.)



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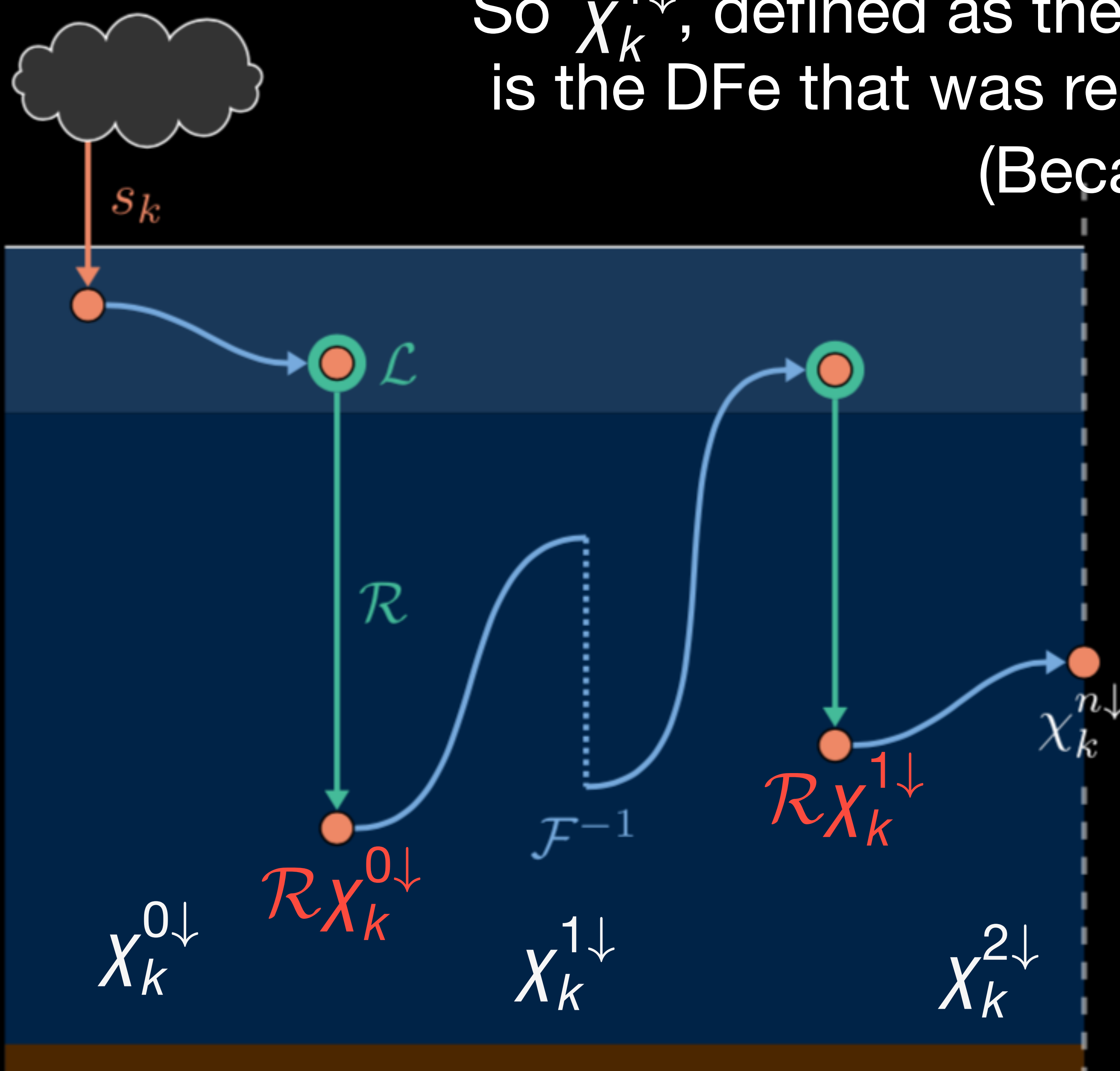
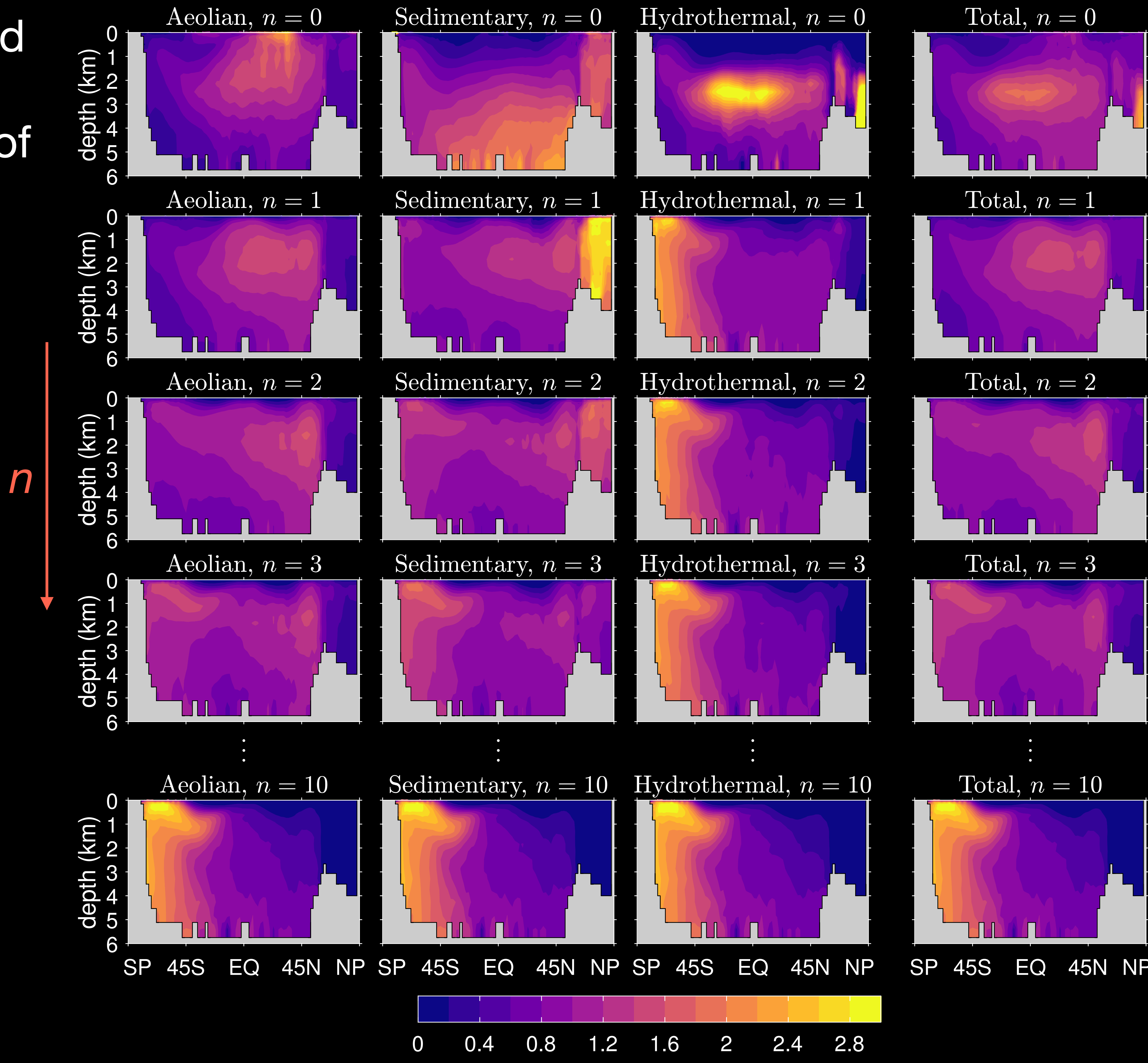


Similarly, $\mathcal{R}\chi_k^{n\downarrow}$ is the source of DFe regenerated n+1 times. Thus, $\chi_k^{n+1\downarrow}$, the concentration of DFe that was regenerated exactly n+1 times since birth is given by

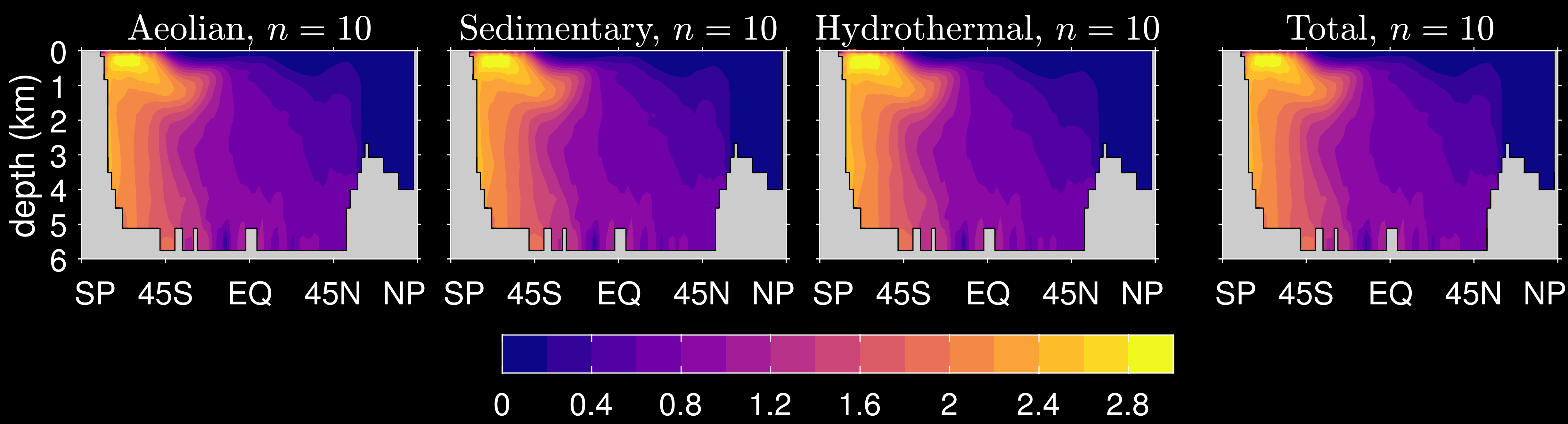
$$\mathcal{F}\chi_k^{(n+1)\downarrow} = \mathcal{R}\chi_k^{n\downarrow} \qquad (2)$$

Normalized zonal averages of $\chi_k^{n\downarrow}(\boldsymbol{r})$



Memory of birth place dissipates quickly

The pattern for $\chi_k^{n\downarrow}(\boldsymbol{r})$ suggest Southern Ocean nutrient trapping:

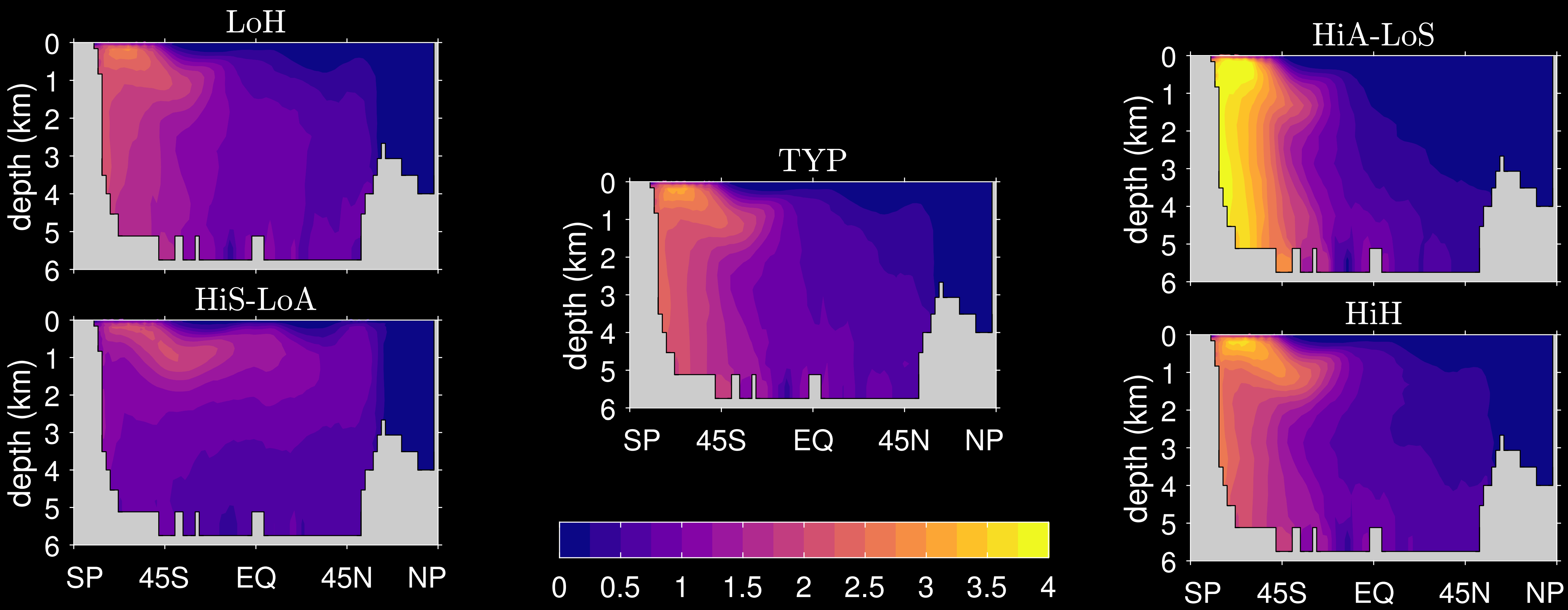


Mathematically, the patterns of $\chi_k^{n\downarrow}$ for large n correspond to the eigen mode of the largest eigen value of $\mathcal{A} \equiv \mathcal{F}^{-1}\mathcal{R}$, from

$$\chi_k^{n\downarrow} = \mathcal{A}^n \, \chi_k^{0\downarrow}$$

Independent of source type *k*!

How robust is the eigen pattern to source scenario?



		Iron sources [Gmol Fe yr-1]			
State estimate	Acronym	σ_A	σ_S	σ_H	σ_{tot}
Low-hydrothermal	LoH	5.3	1.7	0.15	7.1
High-sedimentary-low-aeolian	HiS-LoA	1.8	6.2	0.87	8.9
Typical	TYP	5.3	1.7	0.88	7.9
High-aeolian-low-sedimentary	HiA-LoS	15	0.45	0.88	16
High-hydrothermal	HiH	6.3	2.0	2.3	11

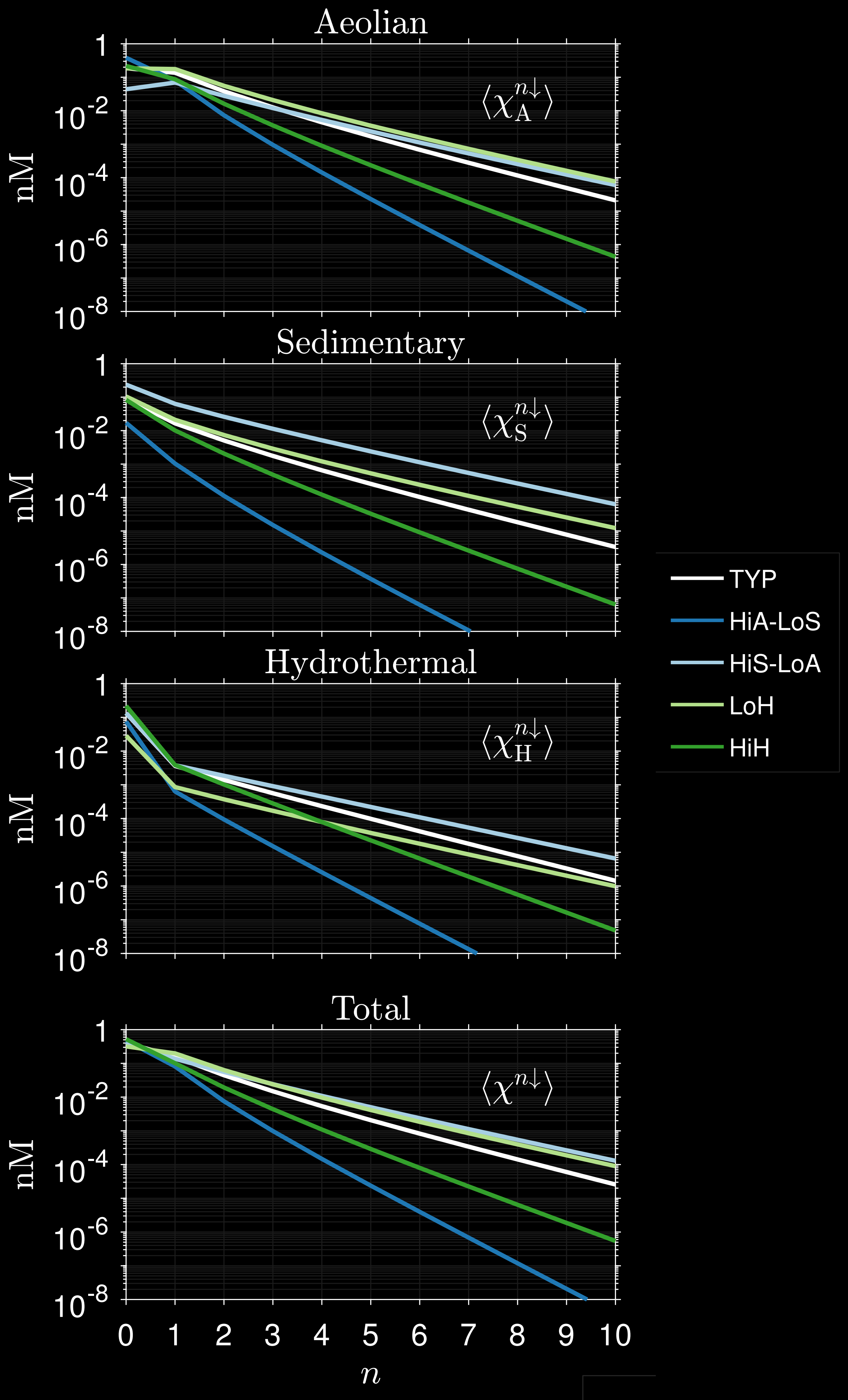
Magnitude decreases exponentially with *n*

$$\lambda^n = \exp(-n/n^*)$$

where λ is the largest eigen value of $\mathcal{A}$. And

$$n^* = -1/\log(\lambda)$$

[*Pasquier and Holzer*, under review]



$\bar{n}_k$, the mean number of past regenerations

$$\begin{cases} \mathcal{F}\chi_k^{0\downarrow} = s_k & (1) \\ \mathcal{F}\chi_k^{(n+1)\downarrow} = \mathcal{R}\chi_k^{n\downarrow} & (2) \end{cases}$$

$$\bar{n}_k \equiv \sum_{n=0}^{\infty} n \frac{\chi_k^{n\downarrow}}{\chi_k}$$

After some math, one can show that $\bar{n}_k$ obeys

$$\mathcal{H}(\bar{n}_k \chi_k) = \mathcal{R}\chi_k$$

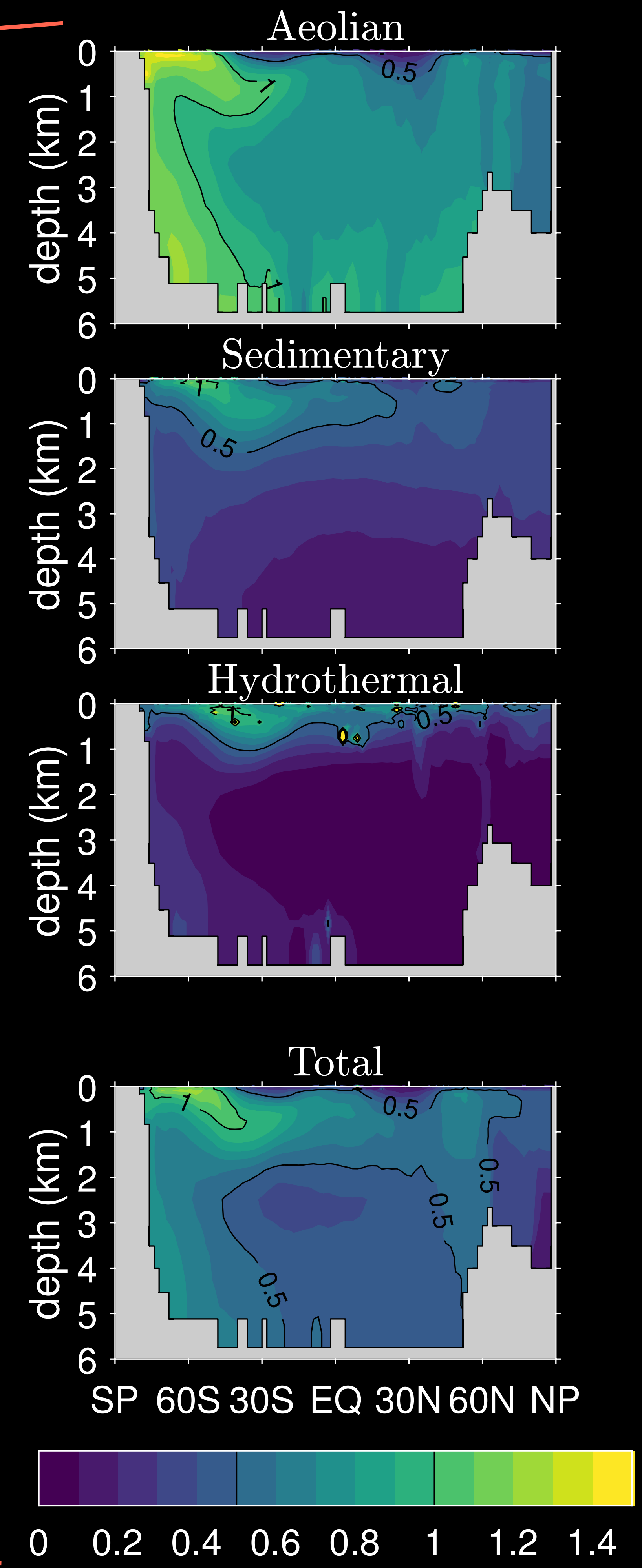
Single matrix inversion!

The mean number $\overline{n}_k$ of regenerations in the past is largest at the surface and in the Southern Ocean

The mean number of phosphorus molecules, $\overline{n}_k^{\mathrm{P}}(\boldsymbol{r})$, globally exported in the past, per DFe molecule that is currently at $\boldsymbol{r}$, obeys

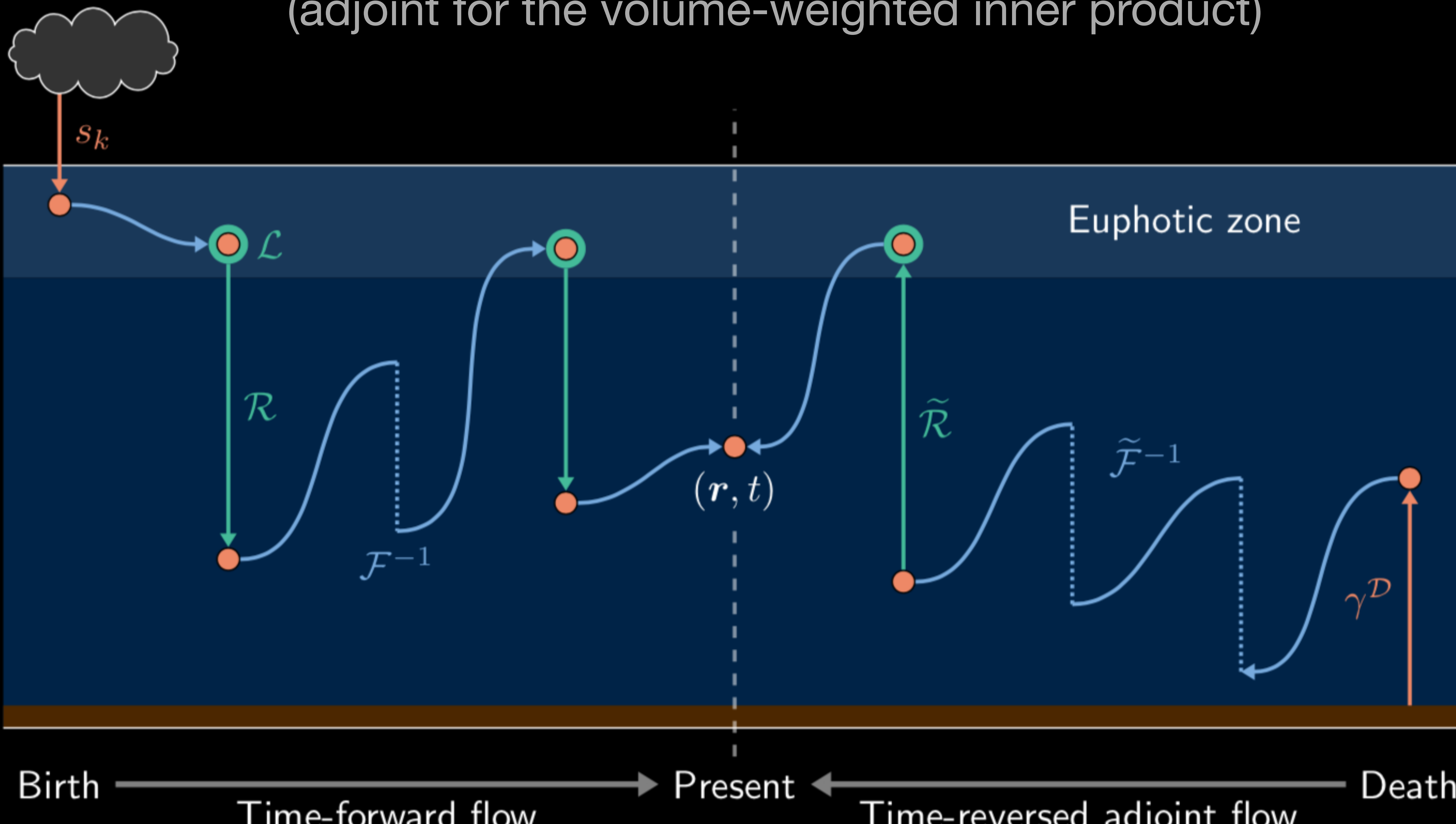
Single matrix inversion!

$$\mathcal{H}(\overline{n}_k^{\mathrm{P}} \chi_k) = \mathcal{R}^{\mathrm{P}} \chi_k$$



Partitioning DFe by *m*, the number of future regenerations

Use the adjoint operators
(adjoint for the volume-weighted inner product)



Partitioning DFe by *m*, the number of future regenerations

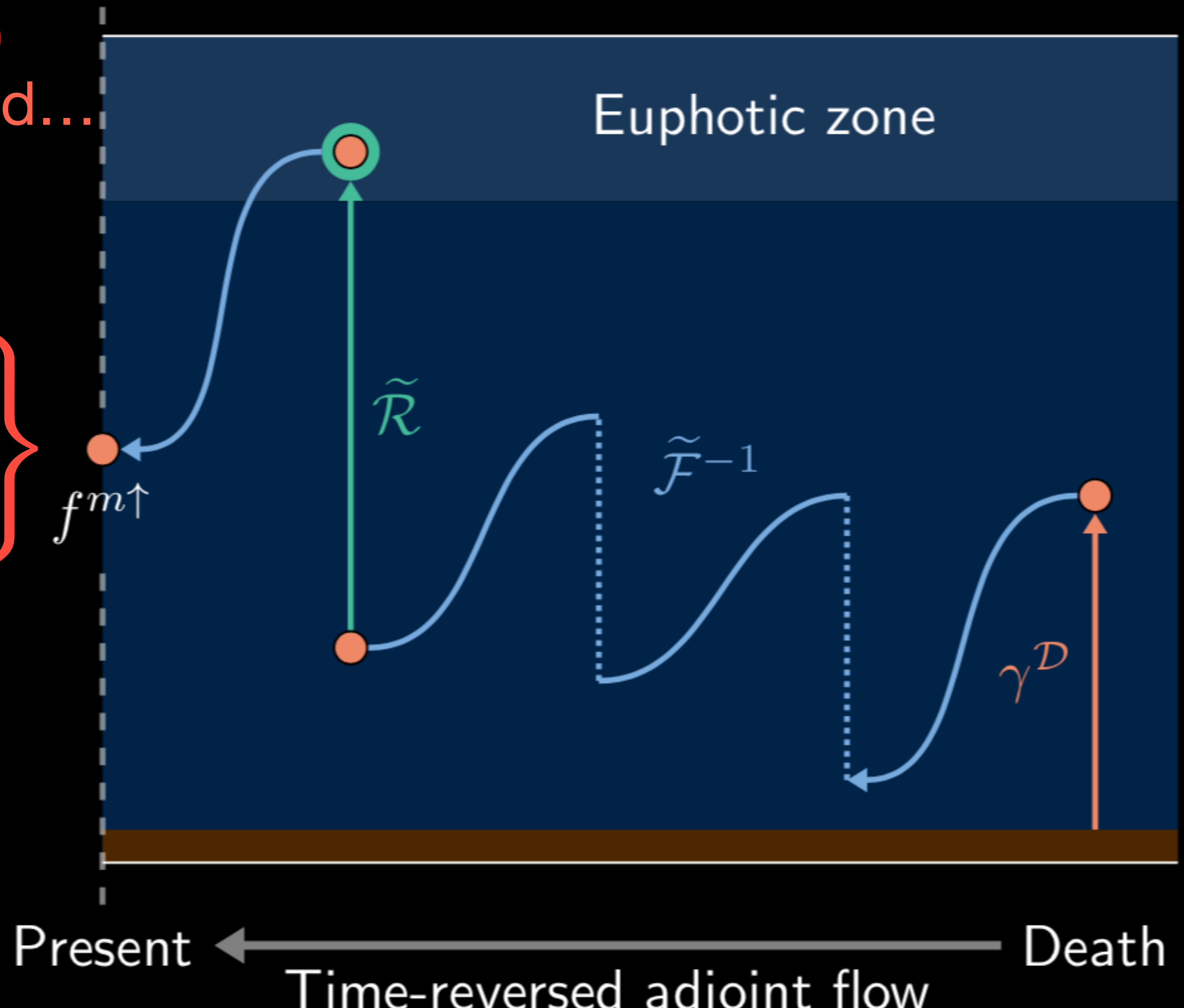
$\gamma^{\mathcal{D}}$ is the specific death rate (i.e., DFe scavenged and buried)

The fraction of DFe $f^{0\uparrow}(\boldsymbol{r})$ that will not be regenerated…

Recursion:

$$\widetilde{\mathcal{F}} f^{0\uparrow} = \gamma^{\mathcal{D}}. \qquad (1)$$

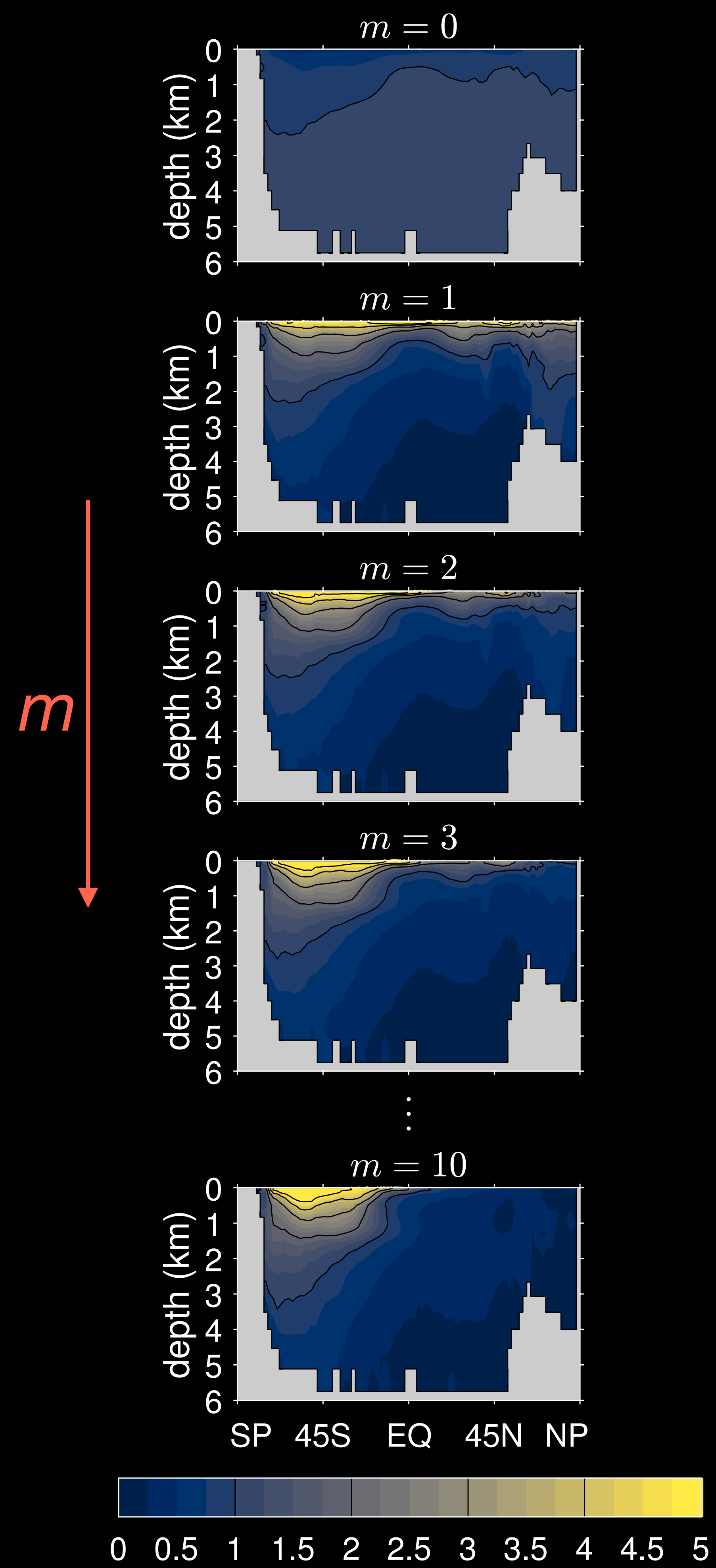
$$\widetilde{\mathcal{F}} f^{(m+1)\uparrow} = \widetilde{\mathcal{R}} f^{m\uparrow} \qquad (2)$$



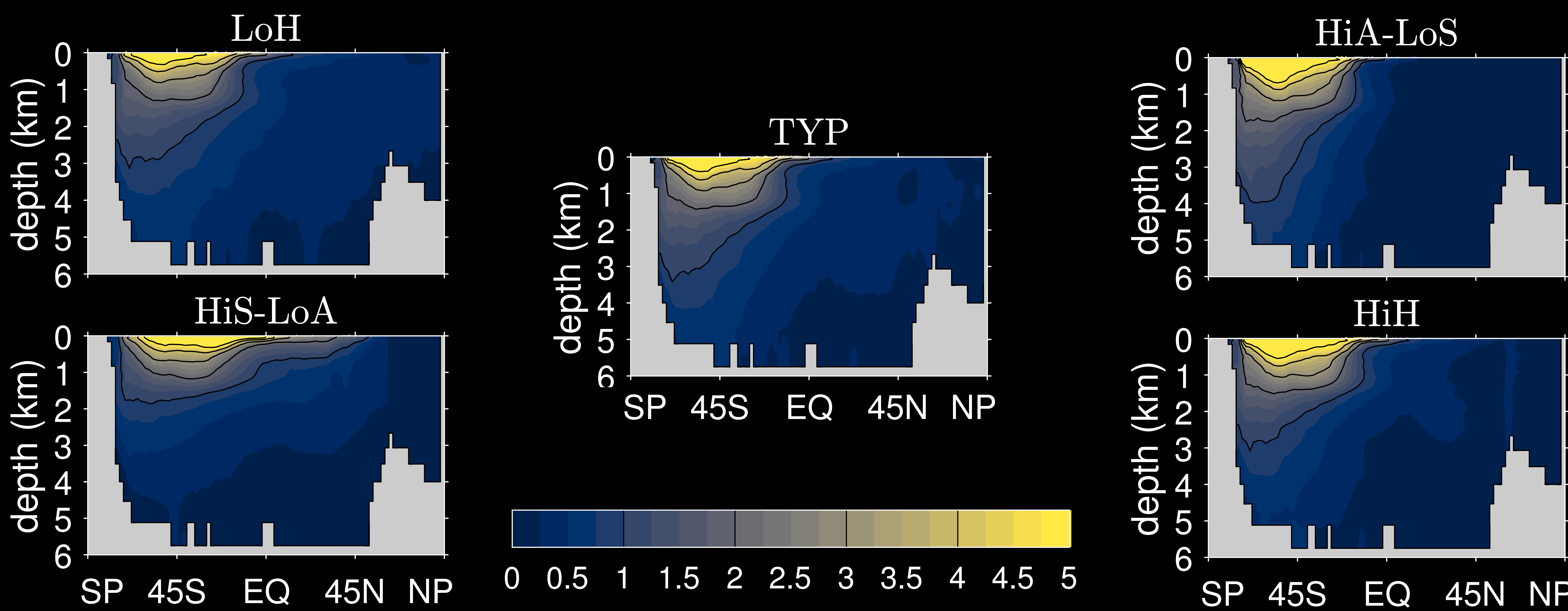
Single matrix inversion!

Normalized zonal averages of $f^{m\uparrow}(\boldsymbol{r})$

- Independent of source type *k*
- Homogeneous for *m*=0, but reaches below 50% of global mean near surface
- Surface concentrated for *m*>0
- Southern-Ocean signature
- Fast convergence to eigen mode



How robust is the eigen pattern to source scenario?



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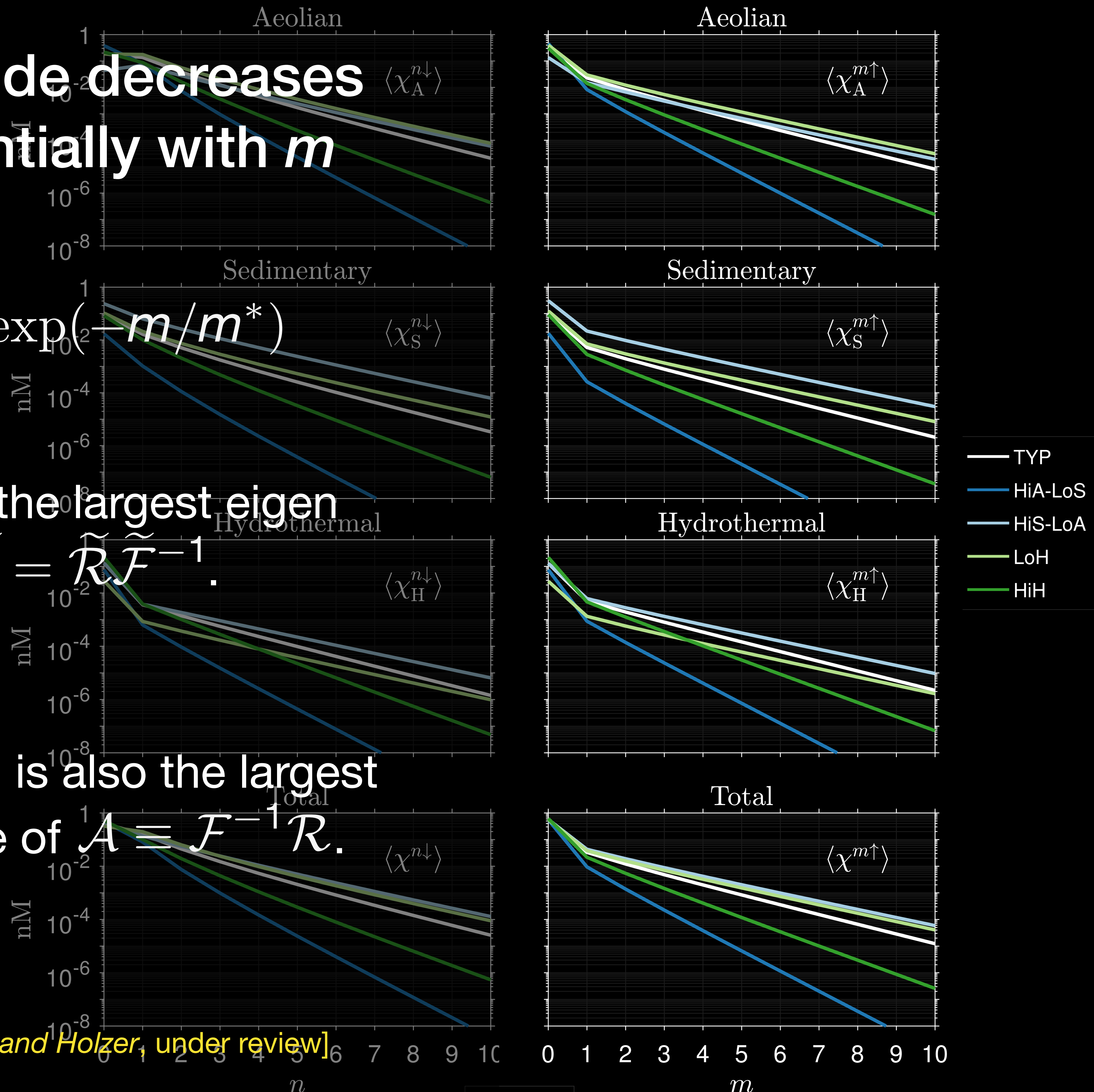
Magnitude decreases exponentially with *m*

$$\lambda^m = \exp(-m/m^*)$$

where λ is the largest eigen value of $\widetilde{\mathcal{A}} = \widetilde{\mathcal{R}}\widetilde{\mathcal{F}}^{-1}$.

Note that λ is also the largest eigen value of $\mathcal{A} = \mathcal{F}^{-1}\mathcal{R}$.



[Pasquier and Holzer, under review]

The mean number of future regenerations:

$$\overline{m}(\boldsymbol{r}) \equiv \sum_{m=0}^{\infty} m\, f^{m\uparrow}(\boldsymbol{r})$$

We can show that $\overline{m}(\boldsymbol{r})$ obeys

$$\widetilde{\mathcal{H}}\,\overline{m} = \widetilde{\mathcal{R}}\, f^{\uparrow}$$

Single matrix inversion!

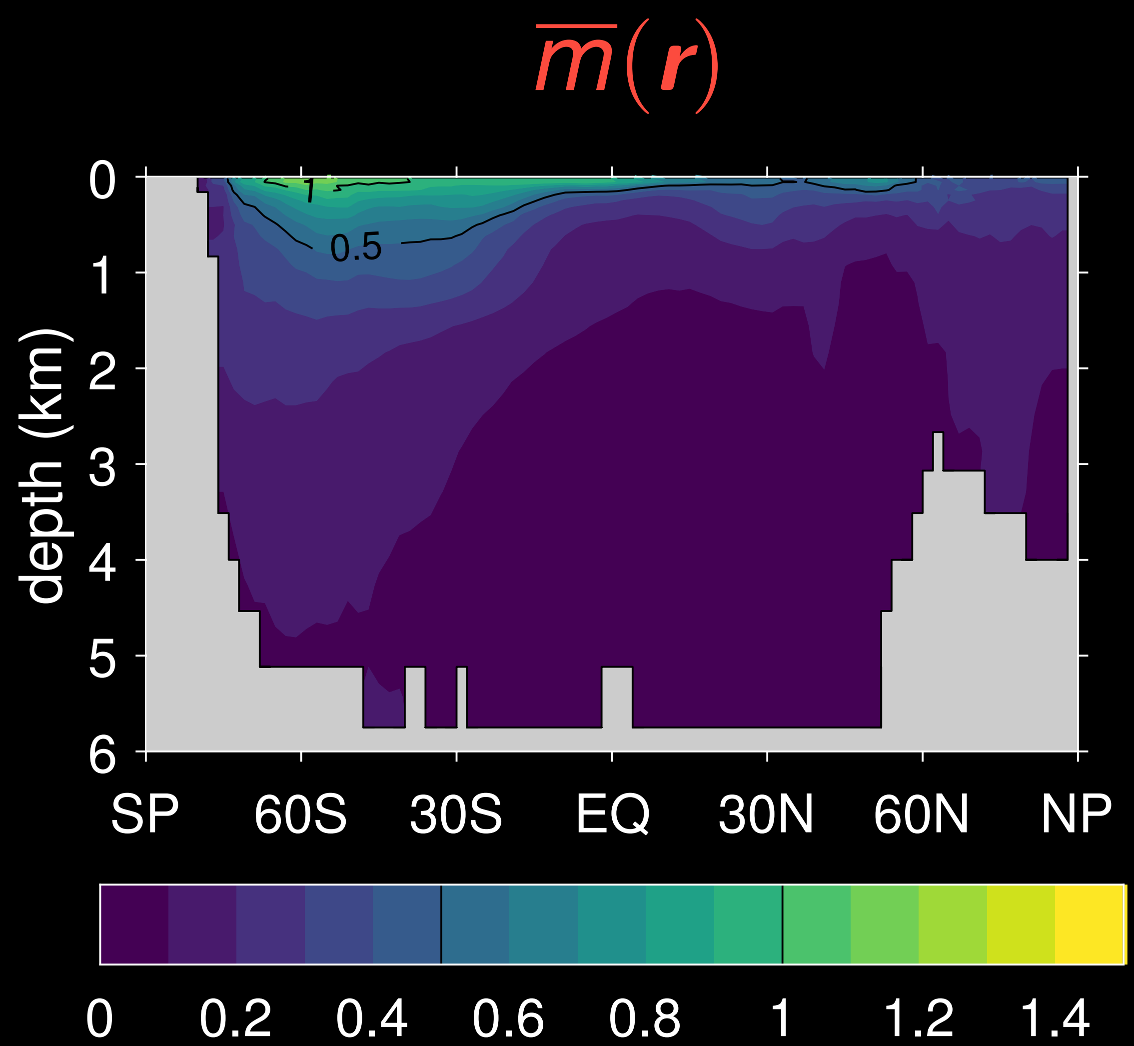
The intrinsic fertilization efficiency:

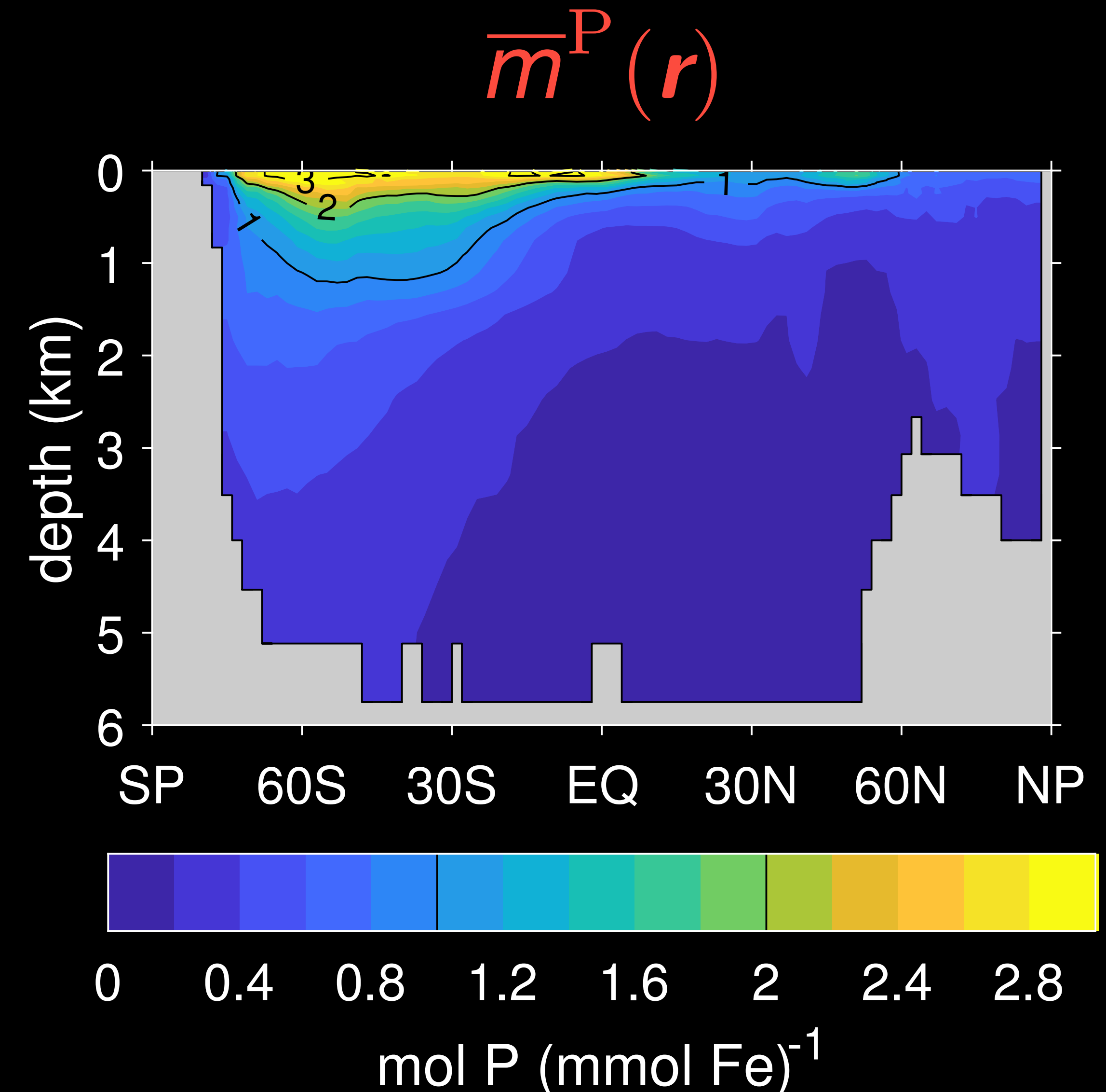
The mean number of phosphorus molecules, $\overline{m}^{\mathrm{P}}(\boldsymbol{r})$, that will be globally exported in the future, per DFe molecule that is currently at $\boldsymbol{r}$, obeys

$$\widetilde{\mathcal{H}}\,\overline{m}^{\mathrm{P}} = \widetilde{\mathcal{R}}^{\mathrm{P}}\, f^{\uparrow}$$

Single matrix inversion!

The intrinsic fertilization efficiency in 3D

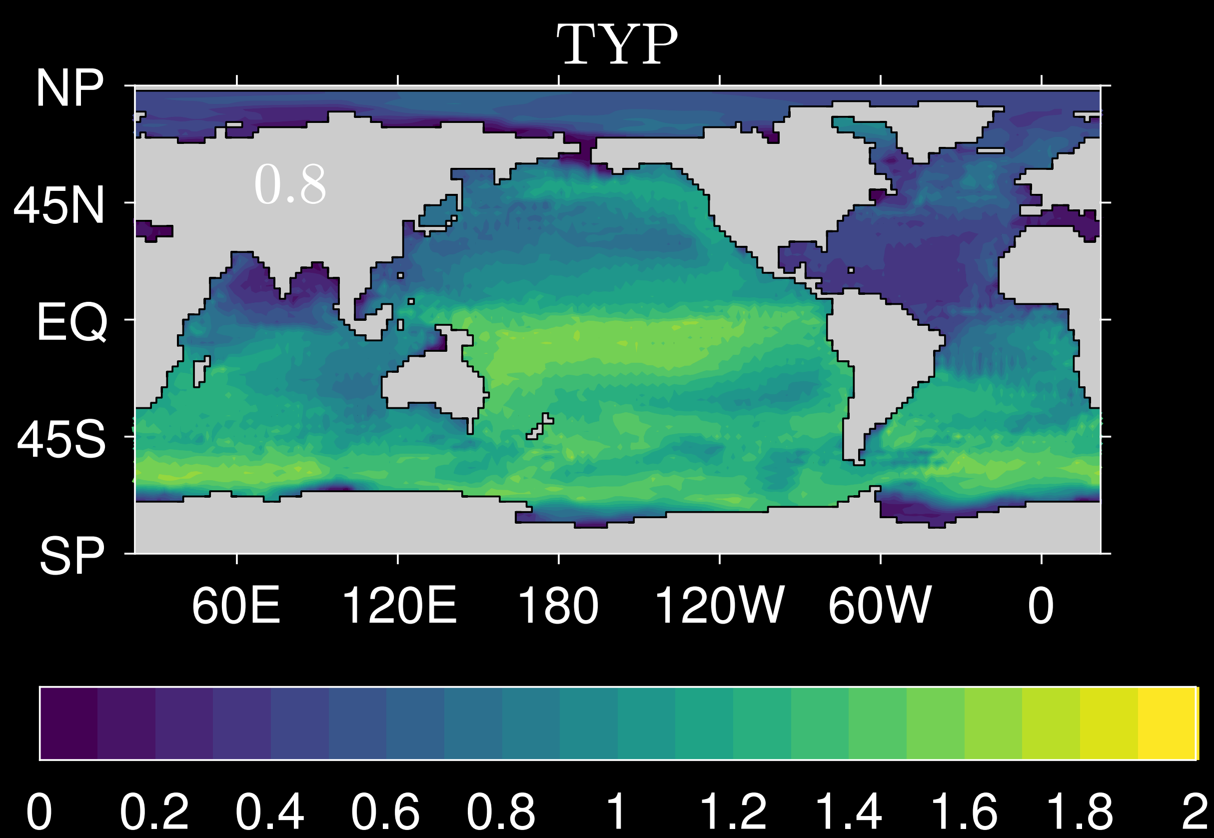




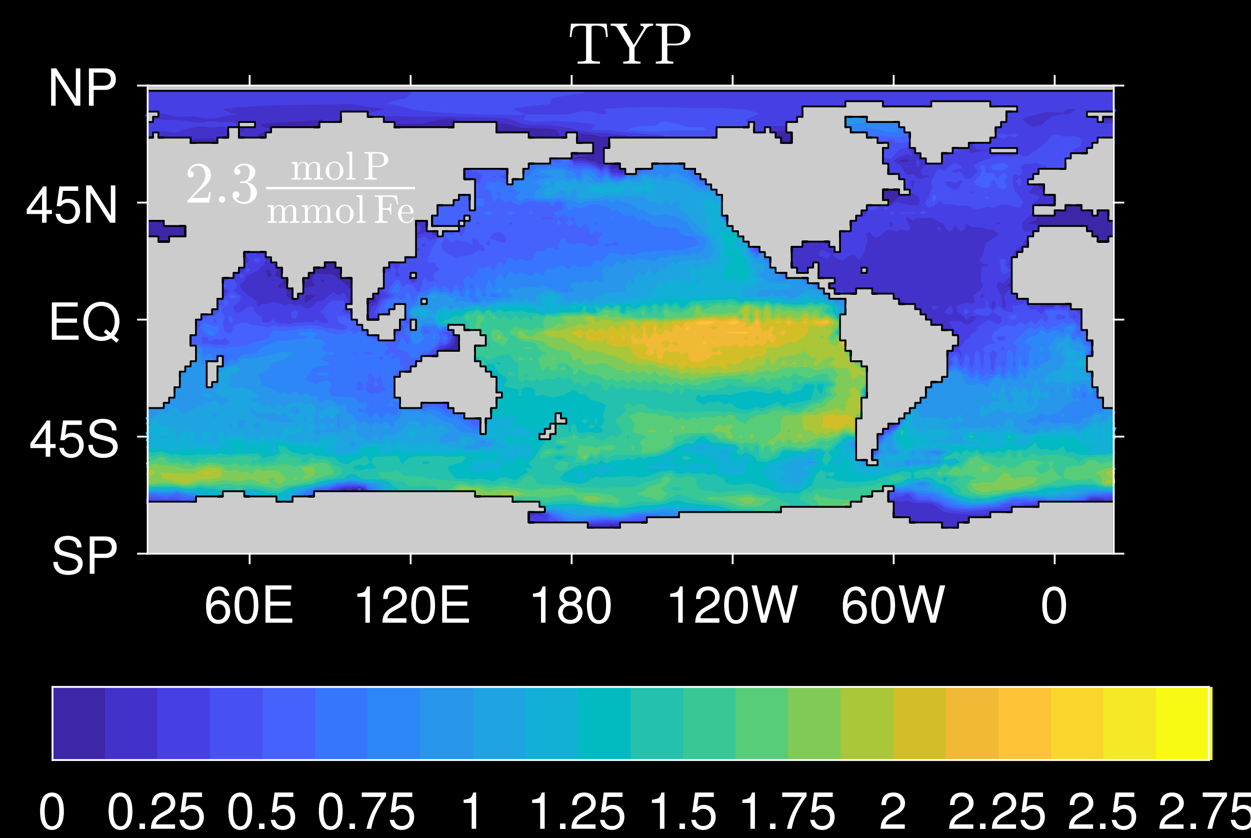
- Independent of source type *k*
- concentrated in the surface
- ZA concentrated in the Southern Ocean

The fertilization pattern at the surface

$\overline{m}(\boldsymbol{r})/\langle\overline{m}\rangle_{\mathrm{surf}}$



$\overline{m}^{\mathrm{P}}(\boldsymbol{r})/\langle\overline{m}^{\mathrm{P}}\rangle_{\mathrm{surf}}$



- Largest in the equatorial Pacific
- Secondary role for the Southern Ocean

Thank you!

Extra slides

Appendix for the math: $\sum x_k^{n\downarrow} = x_k$

Use $x_k^{n\downarrow} = \mathcal{A}^n x_k^{0\downarrow}$ (recall that $\mathcal{A} \equiv \mathcal{F}^{-1}\mathcal{R}$)

$$\sum x_k^{n\downarrow} = \sum \mathcal{A}^n x_k^{0\downarrow}$$

Use $\sum \mathcal{A}^n = (1 - \mathcal{A})^{-1}$

$$\sum x_k^{n\downarrow} = (1 - \mathcal{A})^{-1} x_k^{0\downarrow}$$

Apply $\mathcal{F}(1 - \mathcal{A}) = \mathcal{F} - \mathcal{R} = \mathcal{H}$ to the left

$$\mathcal{H} \sum x_k^{n\downarrow} = \mathcal{F} x_k^{0\downarrow} = s_k$$

So $\sum x_k^{n\downarrow} = x_k$ because they are the solution to the same linear system of equations.

Appendix for the math: Derivation of $\bar{n}_k$

$$\chi_k \bar{n}_k = \sum n \chi_k^{n\downarrow}$$

Use $\chi_k^{n\downarrow} = \mathcal{A}^n \chi_k^{0\downarrow}$ (recall that $\mathcal{A} \equiv \mathcal{F}^{-1}\mathcal{R}$)

$$\chi_k \bar{n}_k = \left(\sum n \mathcal{A}^n\right) \chi_k^{0\downarrow}$$

Use $\sum n \mathcal{A}^n = (1 - \mathcal{A})^{-1} \mathcal{A} (1 - \mathcal{A})^{-1}$

And $\chi_k^{0\downarrow} = \mathcal{F}^{-1} s_k = \mathcal{F}^{-1}\mathcal{H}\mathcal{H}^{-1} s_k = (1 - \mathcal{A})\chi_k$

with $\mathcal{F}(1 - \mathcal{A}) = \mathcal{F} - \mathcal{R} = \mathcal{H}$

$$\chi_k \bar{n}_k = (1 - \mathcal{A})^{-1} \mathcal{A} \chi_k$$

Multiply on the left by $\mathcal{R}\mathcal{A}^{-1}(1 - \mathcal{A}) = \mathcal{F} - \mathcal{R} = \mathcal{H}$

$$\mathcal{H}(\chi_k \bar{n}_k) = \mathcal{R}\chi_k$$

For realistic source cases, most DFe was not regenerated

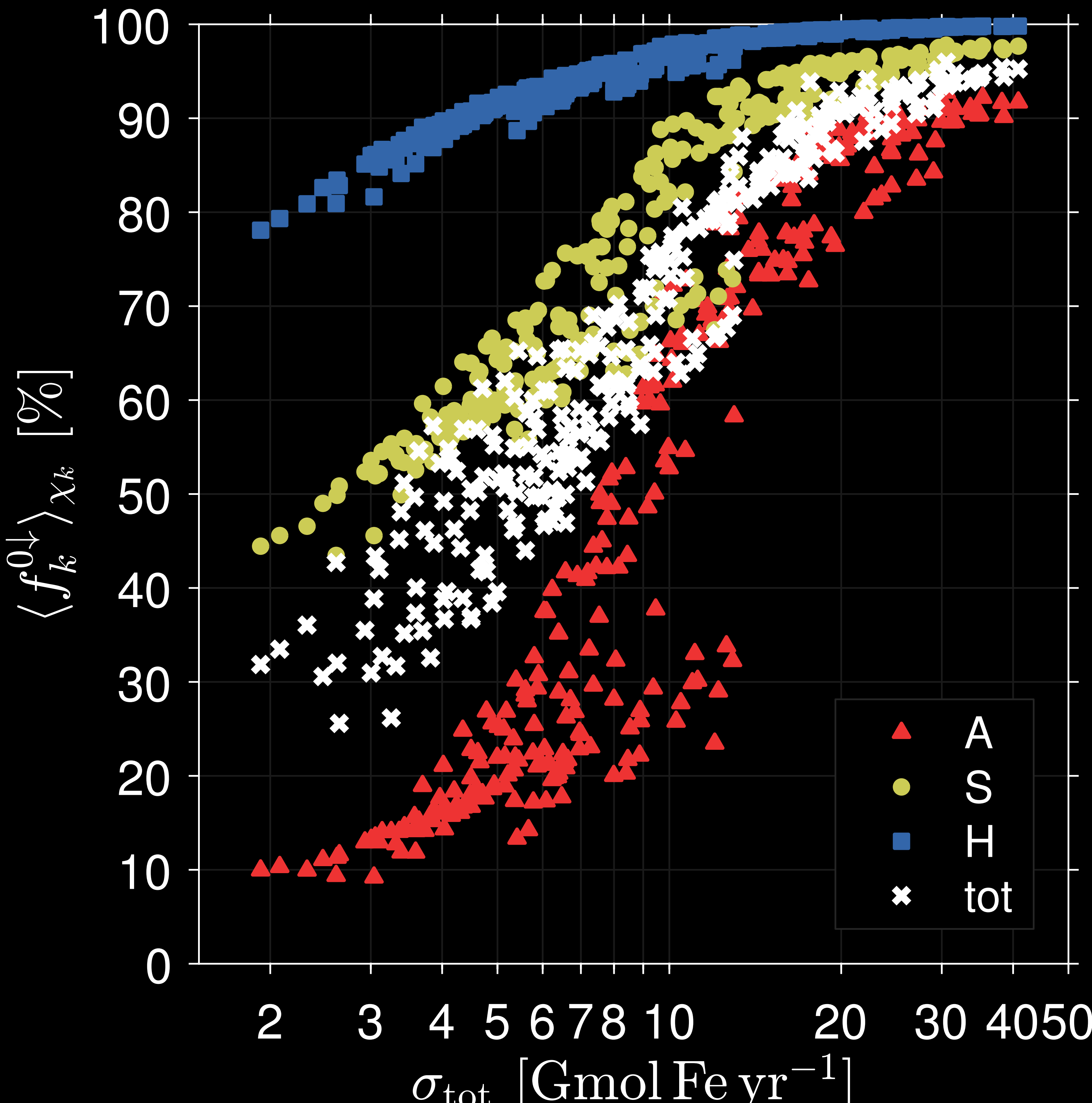
Fraction of DFe that was not regenerated since birth:

$$\langle f_k^{0\downarrow} \rangle_{\chi_k} = \frac{\int \mathrm{d}^3 \boldsymbol{r} f_k^{0\downarrow}(\boldsymbol{r}) \chi_k(\boldsymbol{r})}{\int \mathrm{d}^3 \boldsymbol{r}\, \chi_k(\boldsymbol{r})}$$

where

$$f_k^{0\downarrow} \equiv \chi_k^{0\downarrow} / \chi_k$$

is the local fraction of DFe not regenerated in the past.



How do the magnitudes of $\overline{n}_k$, $\overline{n}_k^{\mathrm{P}}$, $\overline{m}$, and $\overline{m}^{\mathrm{P}}$ depend on source scenario?