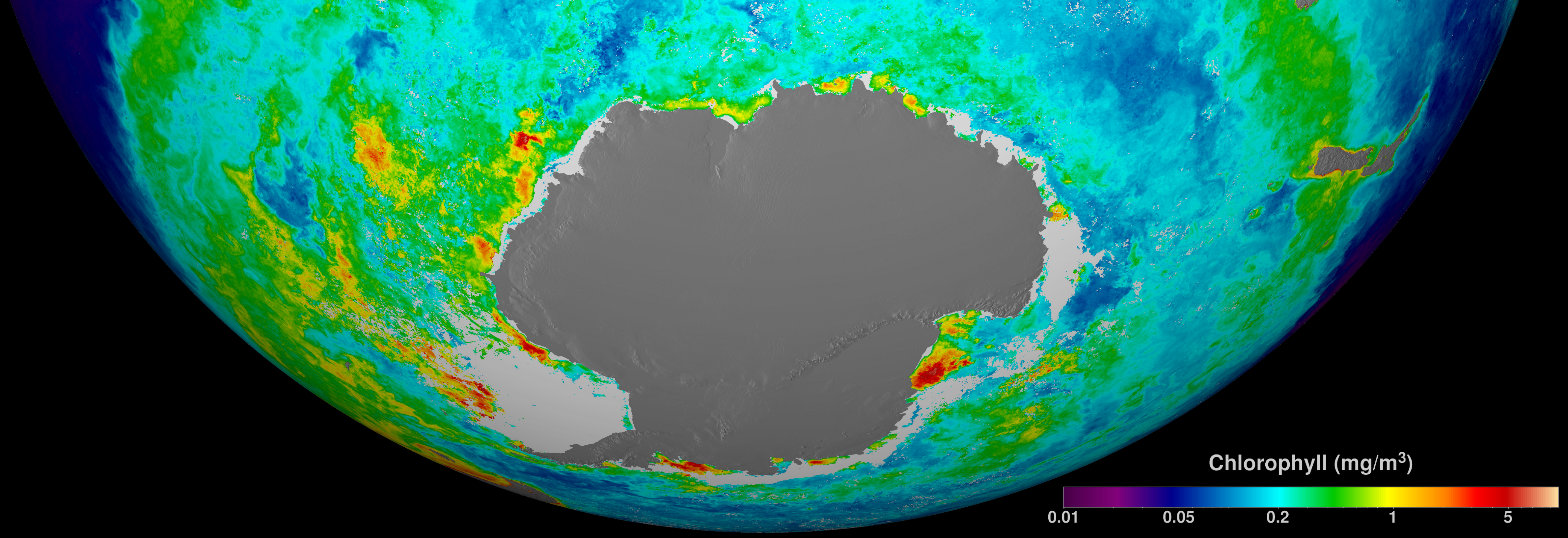
"Give me a half tanker of iron, and I will give you an ice age."

– John Martin, 1988, during at a lecture at WHOI



The efficiency of different iron sources in supporting the ocean's global biological pump

Benoit Pasquier and Mark Holzer

HalfBaked plan

- Estimates of the efficiency of Fe sources in supporting biological production

HalfBaked plan

- Inverse model: estimates of the coupled marine Fe, P, Si cycles
- Estimates of the efficiency of Fe sources in supporting biological production

HalfBaked plan

- Inverse model: estimates of the coupled marine Fe, P, Si cycles

  Maths: Newton solver

- Estimates of the efficiency of Fe sources in supporting biological production

  Maths: non-invasive diagnosis

FePSi model: P and Si cycles

Example with $x_\mathrm{P} = [\mathrm{PO_4}]$ (same with $x_\mathrm{Si} = [\mathrm{Si(OH)_4}]$)

The tracer equation is reorganized in matrix form:

$$(\partial_t + T)x_\mathrm{P} = SU_\mathrm{P} - U_\mathrm{P} + (x_\mathrm{P}^\mathrm{obs} - x_\mathrm{P})/\tau_\mathrm{g}$$

FePSi model: P and Si cycles

Example with $x_\mathrm{P} = [\mathrm{PO_4}]$ (same with $x_\mathrm{Si} = [\mathrm{Si(OH)_4}]$)

The tracer equation is reorganized in matrix form:

Advective–diffusive transport

$$(\partial_t + \boxed{T})x_\mathrm{P} = SU_\mathrm{P} - U_\mathrm{P} + (x_\mathrm{P}^\mathrm{obs} - x_\mathrm{P})/\tau_\mathrm{g}$$

FePSi model: P and Si cycles

Example with $x_\mathrm{P} = [\mathrm{PO_4}]$ (same with $x_\mathrm{Si} = [\mathrm{Si(OH)_4}]$)

The tracer equation is reorganized in matrix form:

Advective–diffusive transport

$$(\partial_t + T)x_\mathrm{P} = SU_\mathrm{P} - U_\mathrm{P} + (x_\mathrm{P}^\mathrm{obs} - x_\mathrm{P})/\tau_\mathrm{g}$$

* Ocean Circulation Inverse Model, pronounced "awesome"

FePSi model: P and Si cycles

Example with $x_\mathrm{P} = [\mathrm{PO}_4]$ (same with $x_\mathrm{Si} = [\mathrm{Si(OH)}_4]$)

The tracer equation is reorganized in matrix form:

Advective–diffusive transport

Uptake rate $U_\mathrm{P}(x_\mathrm{P}, x_\mathrm{Si}, x_\mathrm{Fe})$

$$(\partial_t + T)x_\mathrm{P} = SU_\mathrm{P} - U_\mathrm{P} + (x_\mathrm{P}^\mathrm{obs} - x_\mathrm{P})/\tau_\mathrm{g}$$

* Ocean Circulation Inverse Model, pronounced “awesome”

FePSi model: P and Si cycles

Example with $x_{\mathrm{P}} = [\mathrm{PO_4}]$ (same with $x_{\mathrm{Si}} = [\mathrm{Si(OH)_4}]$)

The tracer equation is reorganized in matrix form:

Advective–diffusive transport

Uptake rate $U_{\mathrm{P}}(x_{\mathrm{P}}, x_{\mathrm{Si}}, x_{\mathrm{Fe}})$

$$(\partial_t + T)x_{\mathrm{P}} = SU_{\mathrm{P}} - U_{\mathrm{P}} + (x_{\mathrm{P}}^{\mathrm{obs}} - x_{\mathrm{P}})/\tau_{\mathrm{g}}$$

Remineralization rate
(at depth, e.g., Martin curve)

This is OCIM*!

* Ocean Circulation Inverse Model, pronounced "awesome"

FePSi model: P and Si cycles

Example with $x_\mathrm{P} = [\mathrm{PO_4}]$ (same with $x_\mathrm{Si} = [\mathrm{Si(OH)_4}]$)

The tracer equation is reorganized in matrix form:

Advective–diffusive transport

Uptake rate $U_\mathrm{P}(x_\mathrm{P}, x_\mathrm{Si}, x_\mathrm{Fe})$

$$(\partial_t + T)x_\mathrm{P} = SU_\mathrm{P} - U_\mathrm{P} + (x_\mathrm{P}^\mathrm{obs} - x_\mathrm{P})/\tau_\mathrm{g}$$

Geological restoring
(constrain total mass)

Remineralization rate
(at depth, e.g., Martin curve)

This is OCIM*!

* Ocean Circulation Inverse Model, pronounced “awesome”

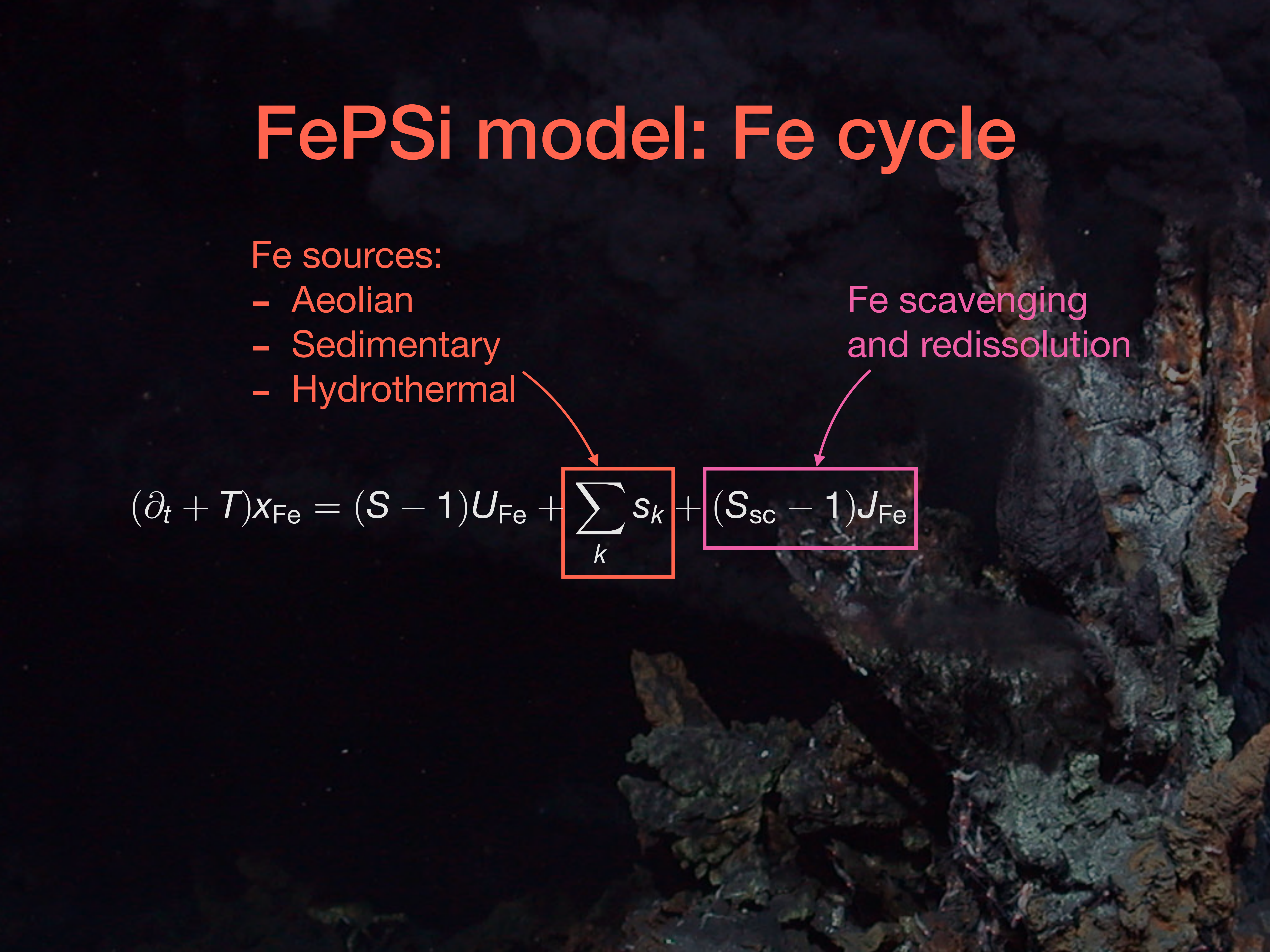
FePSi model: Fe cycle

$$(\partial_t + T)x_{\mathrm{Fe}} = (S - 1)U_{\mathrm{Fe}} + \sum_k s_k + (S_{\mathrm{sc}} - 1)J_{\mathrm{Fe}}$$

FePSi model: Fe cycle

Fe sources:
- Aeolian
- Sedimentary
- Hydrothermal

$$(\partial_t + T)x_{\mathrm{Fe}} = (S - 1)U_{\mathrm{Fe}} + \boxed{\sum_k s_k} + (S_{\mathrm{sc}} - 1)J_{\mathrm{Fe}}$$



FePSi model: Fe cycle

Fe sources:
- Aeolian
- Sedimentary
- Hydrothermal

Fe scavenging and redissolution

$$(\partial_t + T)x_{\mathrm{Fe}} = (S - 1)U_{\mathrm{Fe}} + \boxed{\sum_k s_k} + \boxed{(S_{\mathrm{sc}} - 1)J_{\mathrm{Fe}}}$$

FePSi model: Fe cycle

Fe sources:
- Aeolian
- Sedimentary
- Hydrothermal

Fe scavenging and redissolution

$$(\partial_t + T)x_{\mathrm{Fe}} = (S - 1)U_{\mathrm{Fe}} + \sum_k s_k + (S_{\mathrm{sc}} - 1)J_{\mathrm{Fe}}$$

* Biogeochemical Elemental Cycling

FePSi model: Inverse mode

Model parameters ← optimize ← **Observed data**

solve $\frac{\partial x}{\partial t} = f(x) = 0$

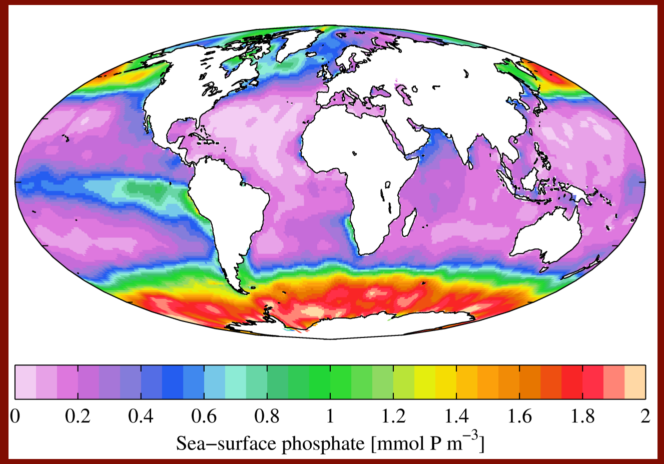
↓

Modelled data

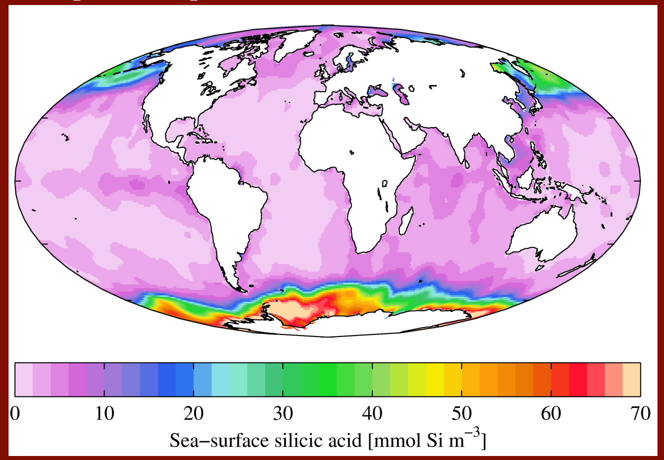
$x =$ dFe, PO_4, $Si(OH)_4$ → compare →

dFe (too sparse)

PO_4:



$Si(OH)_4$:



FePSi model: Inverse mode

Model parameters ← optimize — **Observed data**

solve $\frac{\partial x}{\partial t} = f(x) = 0$

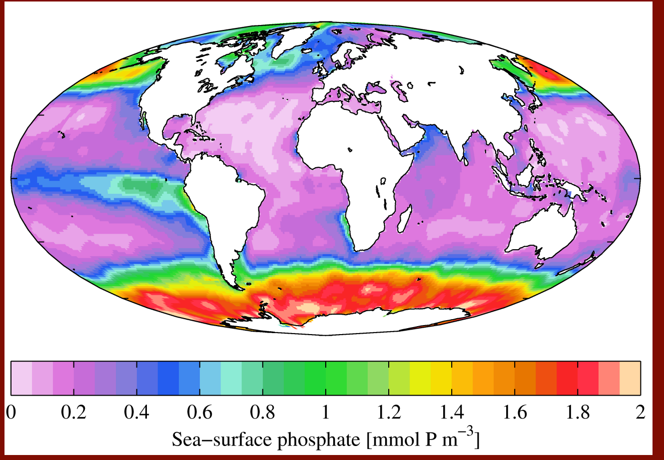
↓

Modelled data

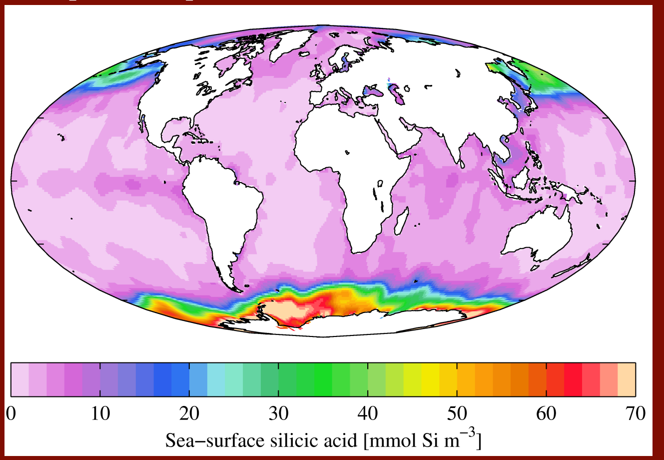
$x =$ dFe, PO_4, $Si(OH)_4$ — compare →

dFe (too sparse)

PO_4:



$Si(OH)_4$:



Solving $f(x) = 0$ is **a lot** faster

than waiting for $\frac{\partial x}{\partial t} = 0$!

FePSi model: Optimization

Concatenate the Fe, P, and Si tracer equations into

$$\partial_t x = f(x) \quad \text{where} \quad x = \begin{bmatrix} x_{\text{P}} \\ x_{\text{Si}} \\ x_{\text{Fe}} \end{bmatrix}$$

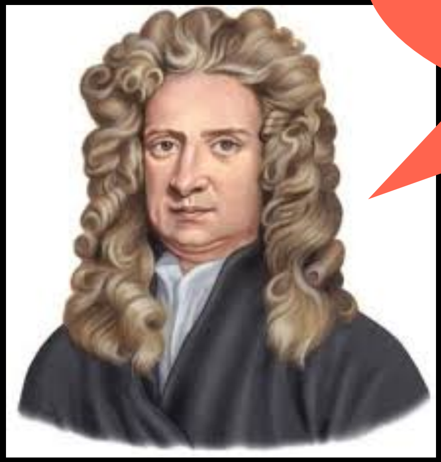
Then solve $\quad f(x) = 0$

FePSi model: Optimization

Concatenate the Fe, P, and Si tracer equations into

$$\partial_t x = f(x) \quad \text{where} \quad x = \begin{bmatrix} x_{\mathrm{P}} \\ x_{\mathrm{Si}} \\ x_{\mathrm{Fe}} \end{bmatrix}$$

Then solve $\quad f(x) = 0$

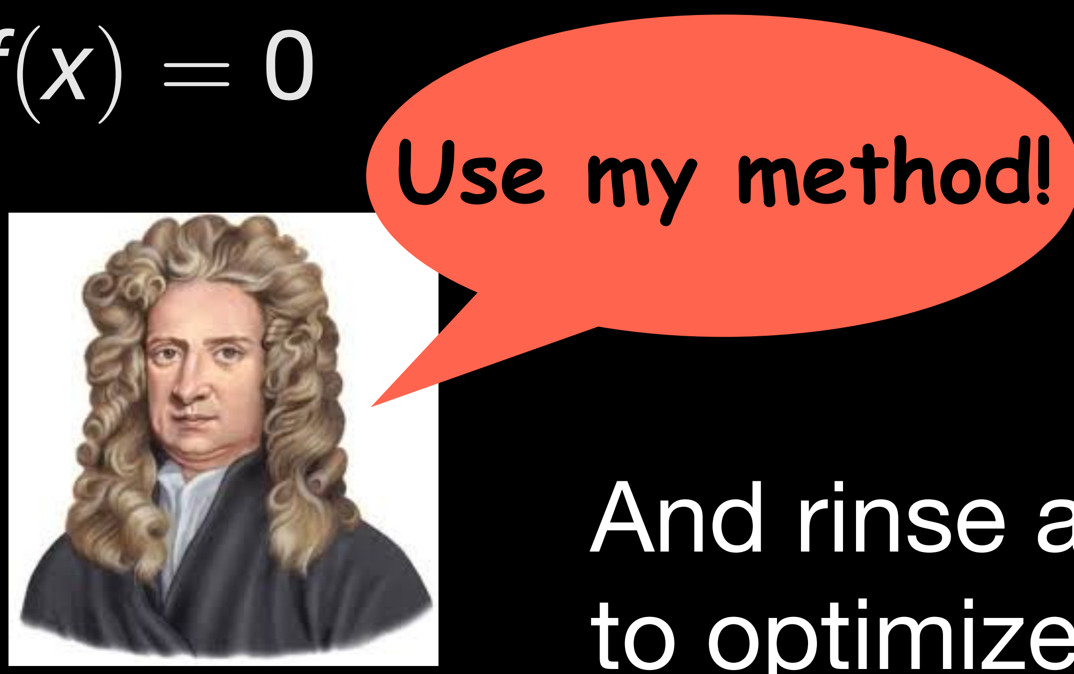


FePSi model: Optimization

Concatenate the Fe, P, and Si tracer equations into

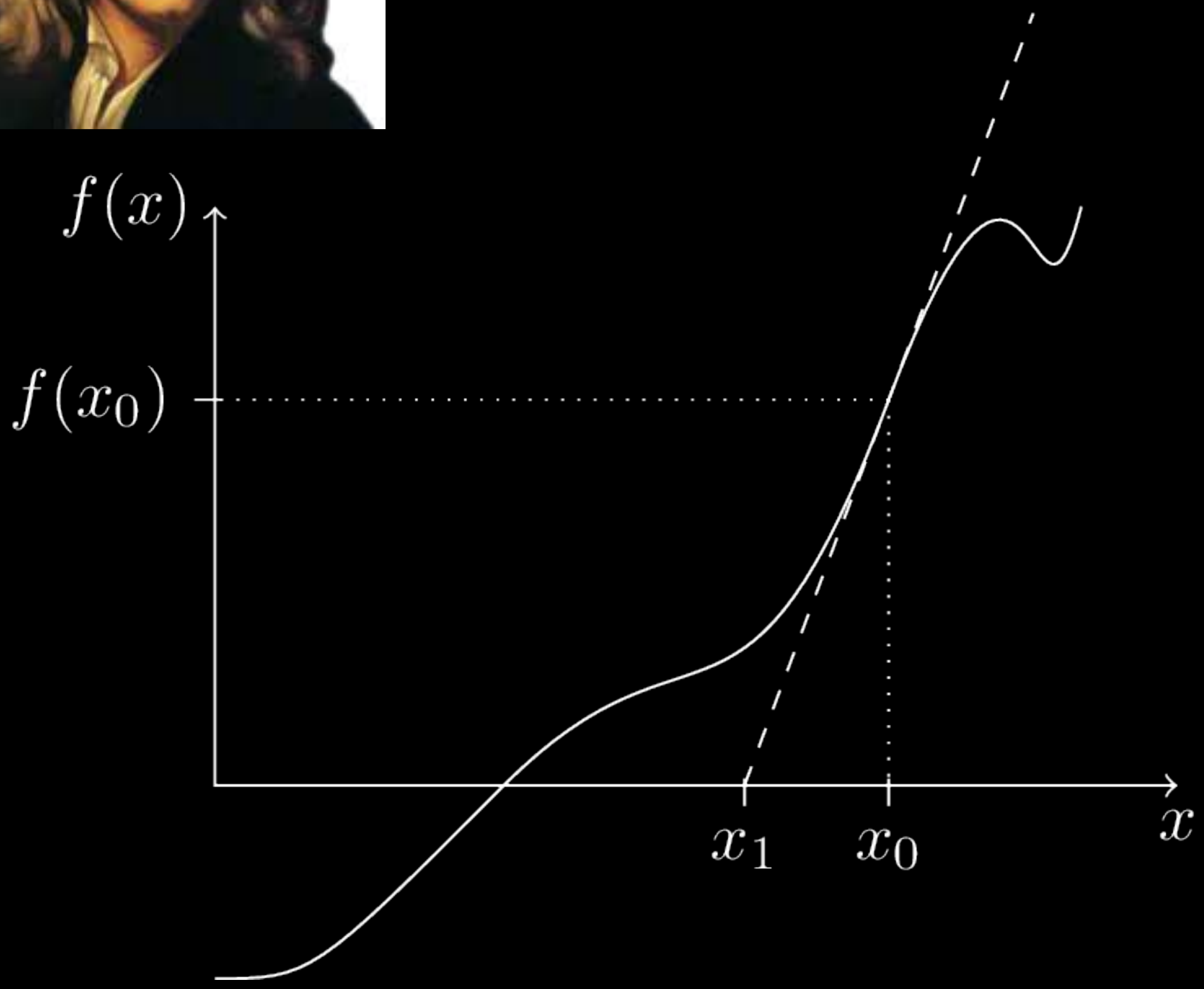
$$\partial_t x = f(x) \quad \text{where} \quad x = \begin{bmatrix} x_{\mathrm{P}} \\ x_{\mathrm{Si}} \\ x_{\mathrm{Fe}} \end{bmatrix}$$

Then solve $f(x) = 0$

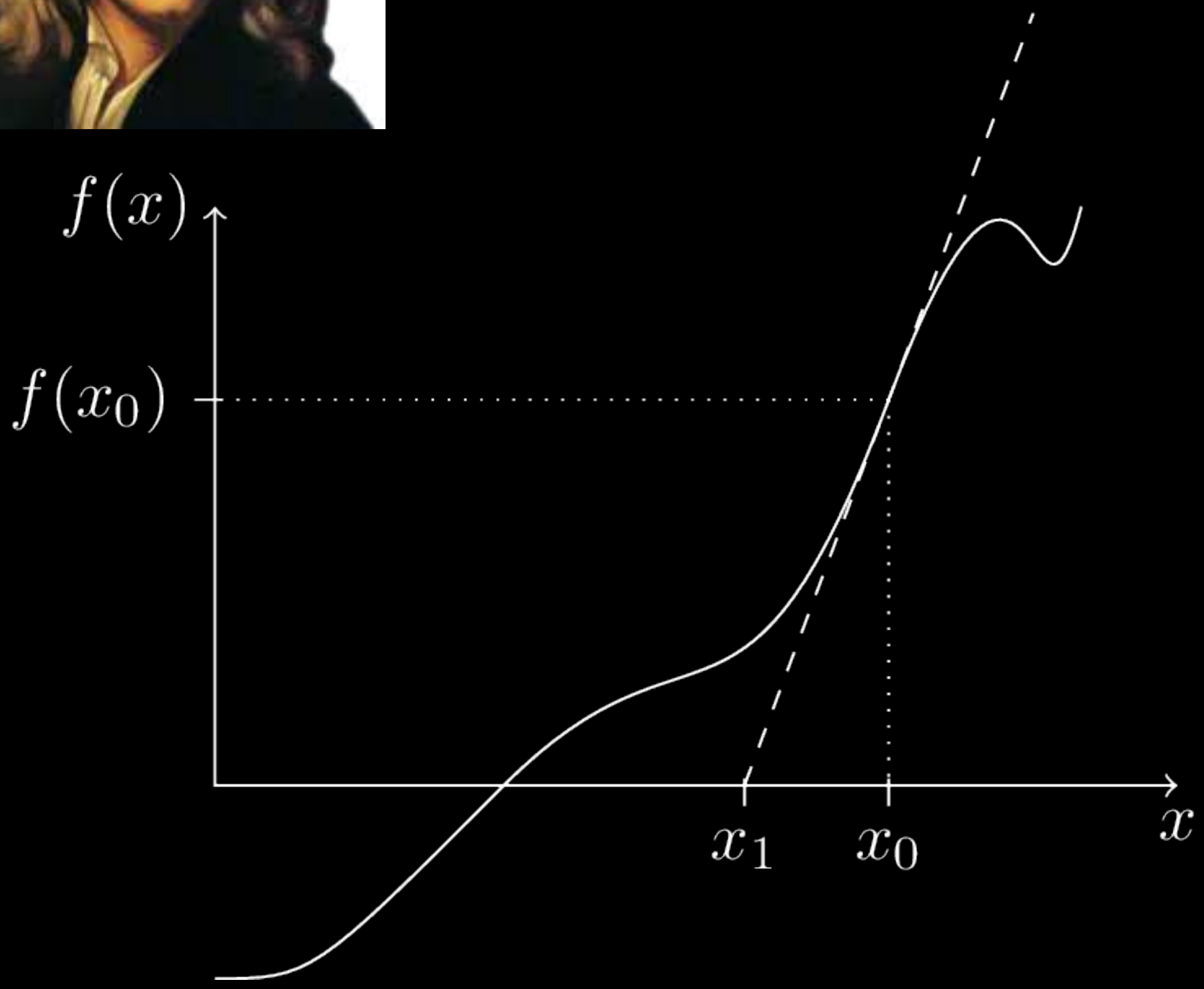


And rinse and repeat
to optimize parameters…

Newton Method



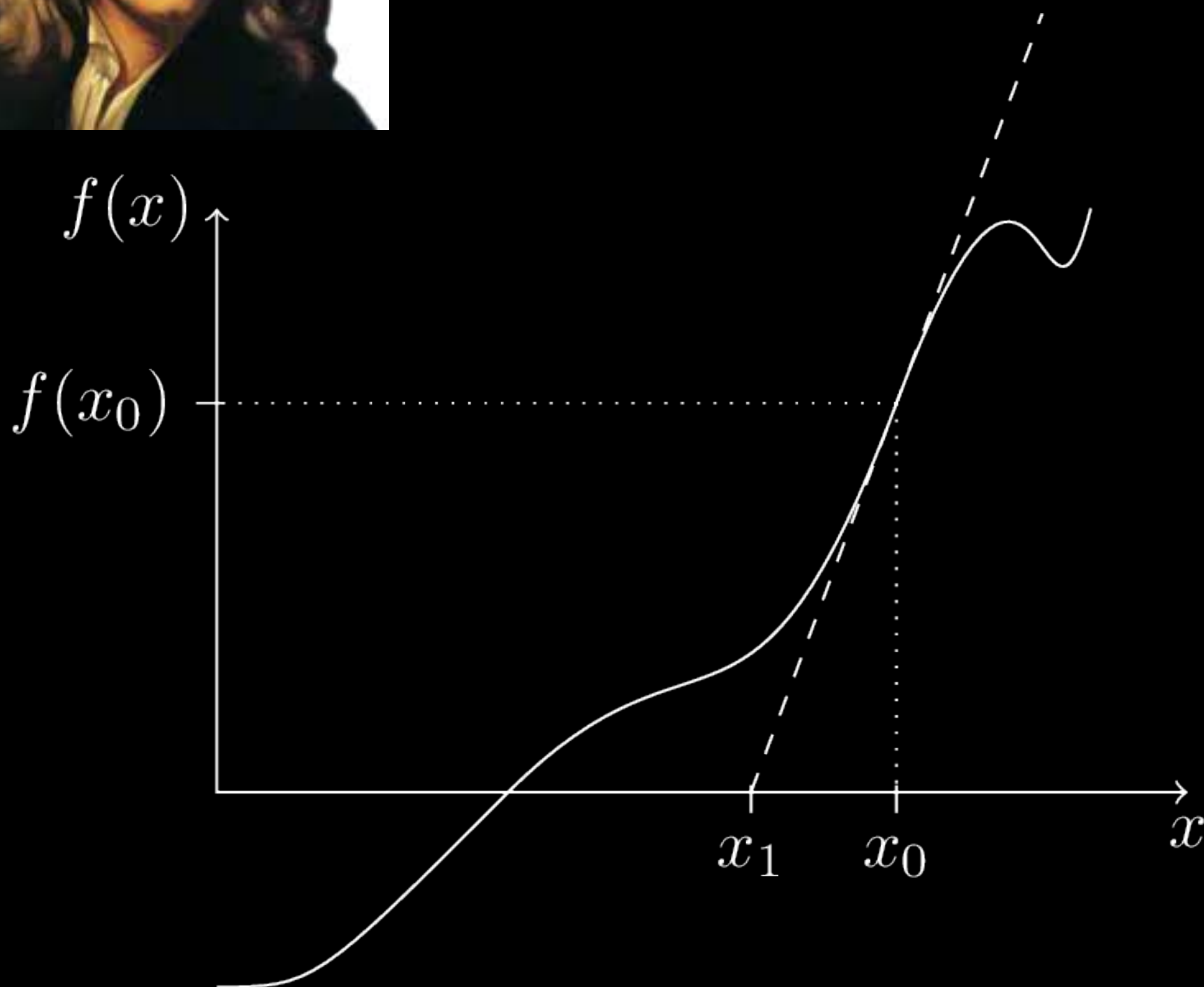
Newton Method



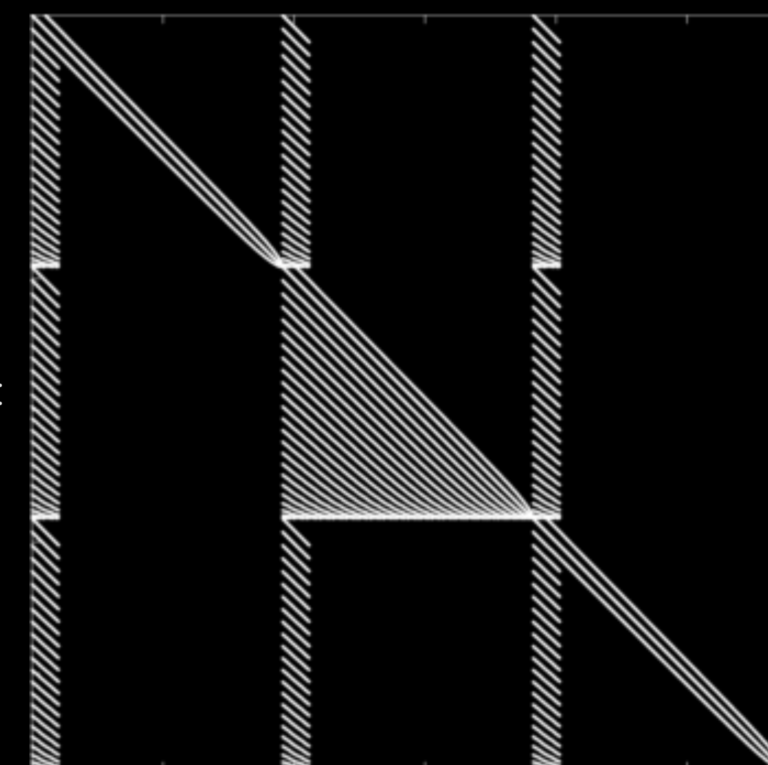
Because $f(x) \simeq f(x_0) + \nabla f(x_0)(x - x_0)$

Then, for $x_1 = x_0 - [\nabla f(x_0)]^{-1} f(x_0)$, $f(x_1) \simeq 0$

Newton Method



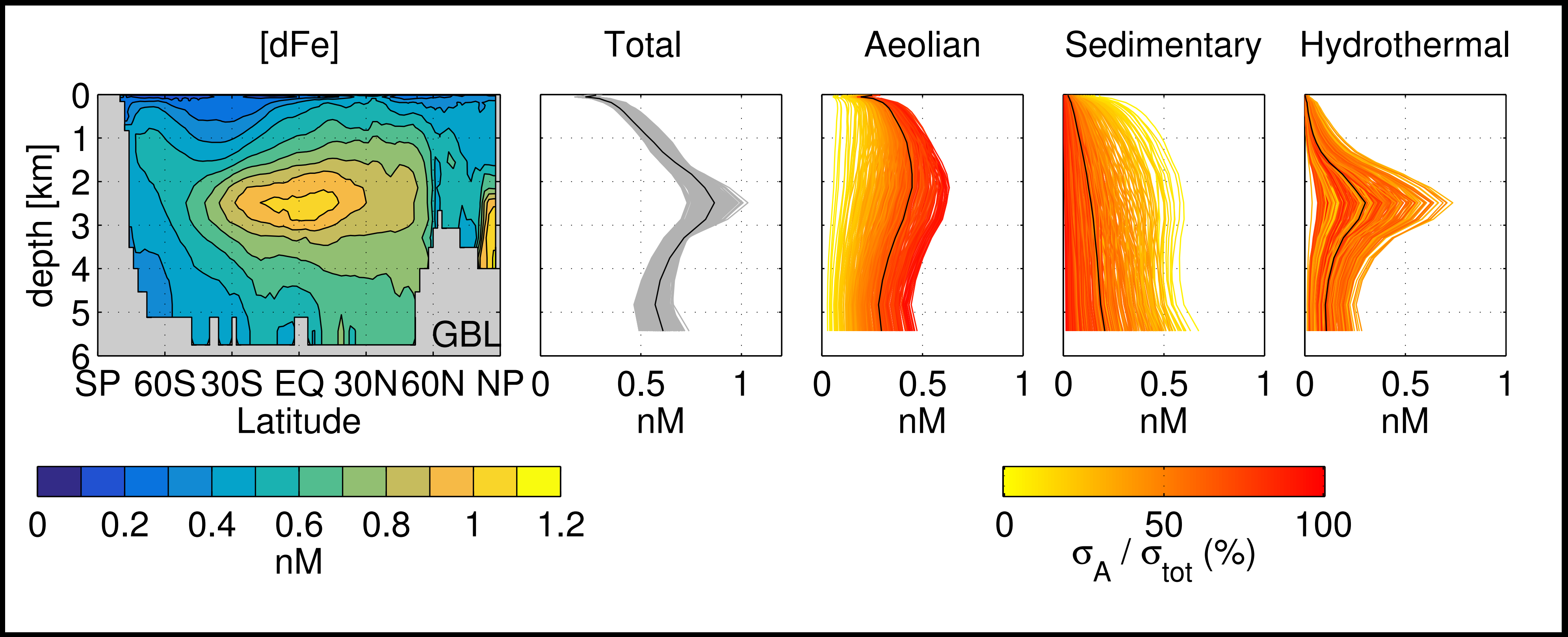
$\nabla f(x) =$



Because $\quad f(x) \simeq f(x_0) + \nabla f(x_0)(x - x_0)$

Then, for $\quad x_1 = x_0 - [\nabla f(x_0)]^{-1} f(x_0) \quad , \quad f(x_1) \simeq 0$

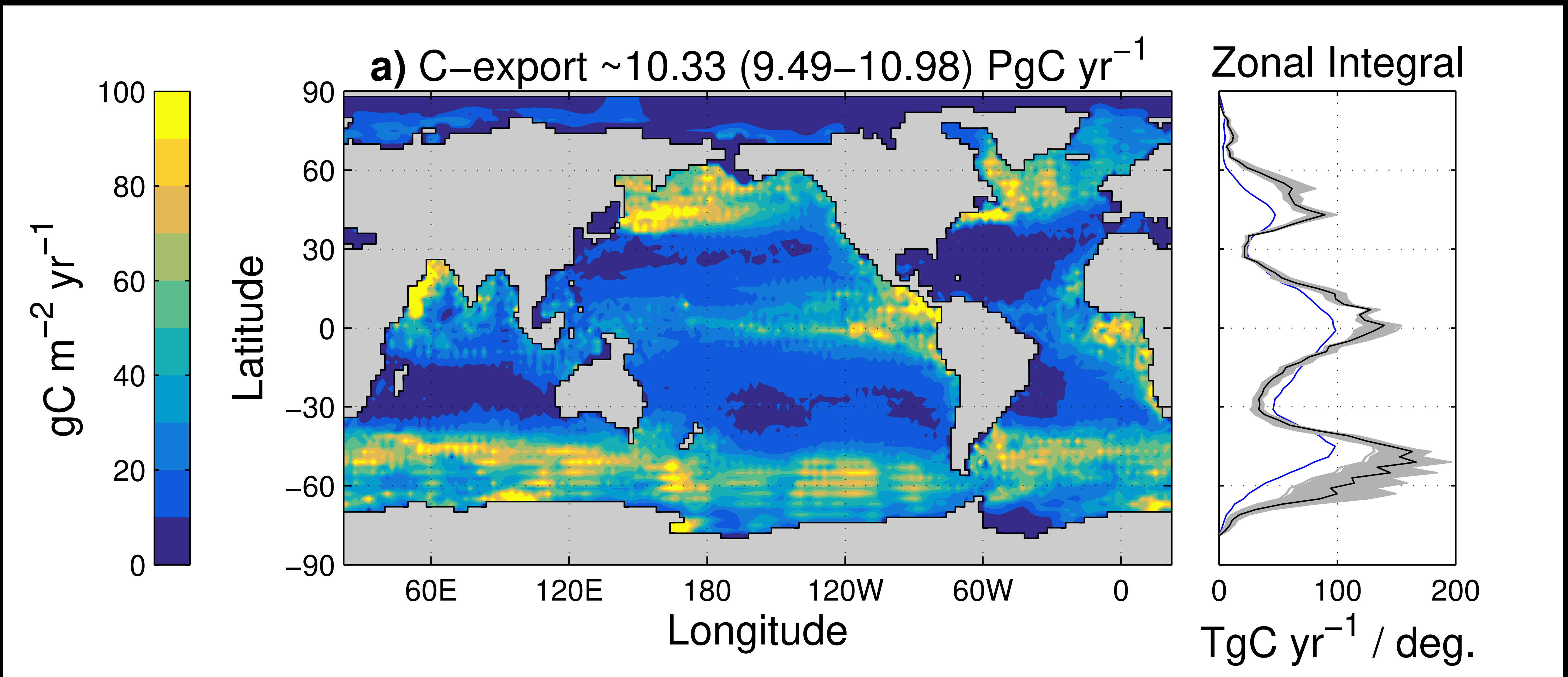
FePSi model: Fe distribution



Sources span 2 orders of magnitude (each), but with the optimization of the sink parameters the total [dFe] are tightly clustered!

All solutions are plausible estimates!

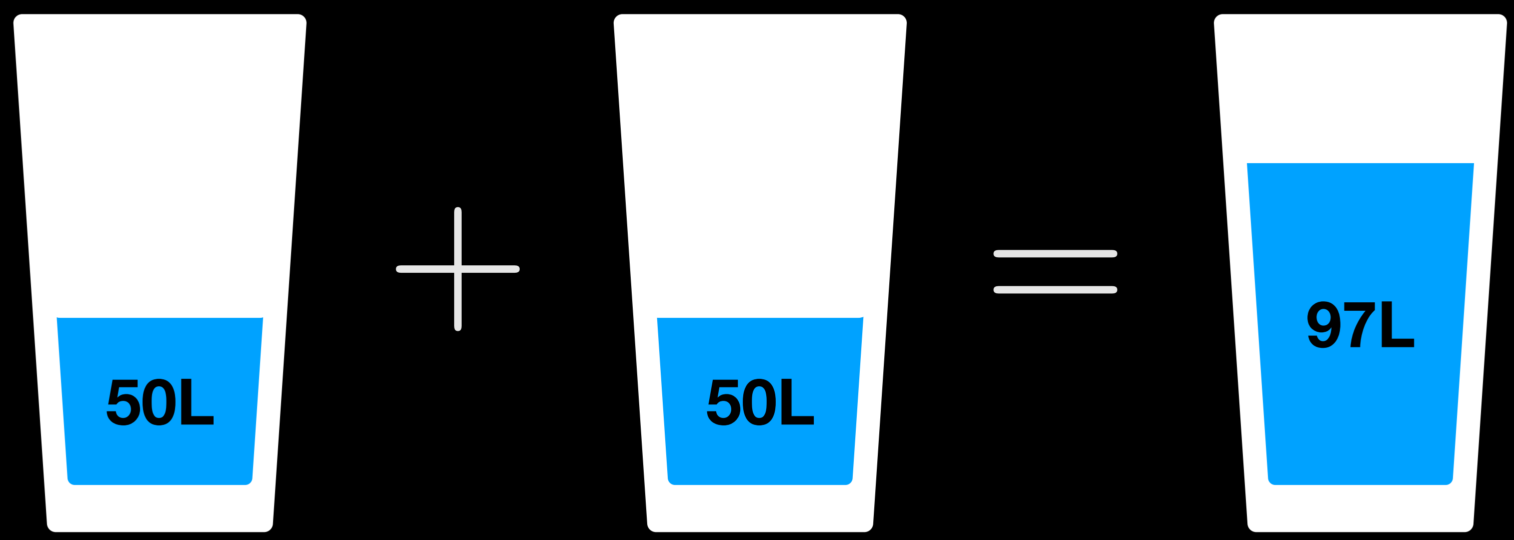
FePSi model: export production



What fraction of export production is due to each Fe source?

Why an equivalent linear model?

Example: the volume contribution of mixed water and ethanol



Standard method is to shutdown other sources, compute the anomaly, and infer the contribution. But this is invasive!

Perturbing the system to estimate an anomaly is a perfectly fine question to ask… But it is not the true contribution in the unperturbed system! (e.g., Holzer et al., 2016)

Equivalent linear model

$$(\partial_t + T)x_{\mathrm{Fe}} = \boxed{(S-1)U_{\mathrm{Fe}}} + \sum_k s_k + \boxed{(S_{\mathrm{sc}}-1)J_{\mathrm{Fe}}}$$

$$L_{U_{\mathrm{Fe}}} = U_{\mathrm{Fe}}/x_{\mathrm{Fe}}$$

$$L_{J_{\mathrm{Fe}}} = J_{\mathrm{Fe}}/x_{\mathrm{Fe}}$$

$$L = T + \boxed{(1-S)L_{U_{\mathrm{Fe}}}} + \boxed{(1-S_{\mathrm{sc}})L_{J_{\mathrm{Fe}}}}$$

Equivalent linear model

$$(\partial_t + T)x_{\mathrm{Fe}} = \boxed{(S-1)U_{\mathrm{Fe}}} + \sum_k s_k + \boxed{(S_{\mathrm{sc}}-1)J_{\mathrm{Fe}}}$$

$$L_{U_{\mathrm{Fe}}} = U_{\mathrm{Fe}}/x_{\mathrm{Fe}}$$

$$L_{J_{\mathrm{Fe}}} = J_{\mathrm{Fe}}/x_{\mathrm{Fe}}$$

$$L = T + \boxed{(1-S)L_{U_{\mathrm{Fe}}}} + \boxed{(1-S_{\mathrm{sc}})L_{J_{\mathrm{Fe}}}}$$

Equivalent linear system: $(\partial_t + L)x_{\mathrm{Fe}} = \sum_k s_k$

Equivalent linear model

$$(\partial_t + L)x_{\mathrm{Fe}} = \sum_k s_k$$

Allows non-invasive estimation of the true contribution of each source (s_k) to the total dFe:

$$x_k = L^{-1}s_k \qquad \text{with} \qquad x_{\mathrm{Fe}} = \sum_k x_k$$

Equivalent linear model

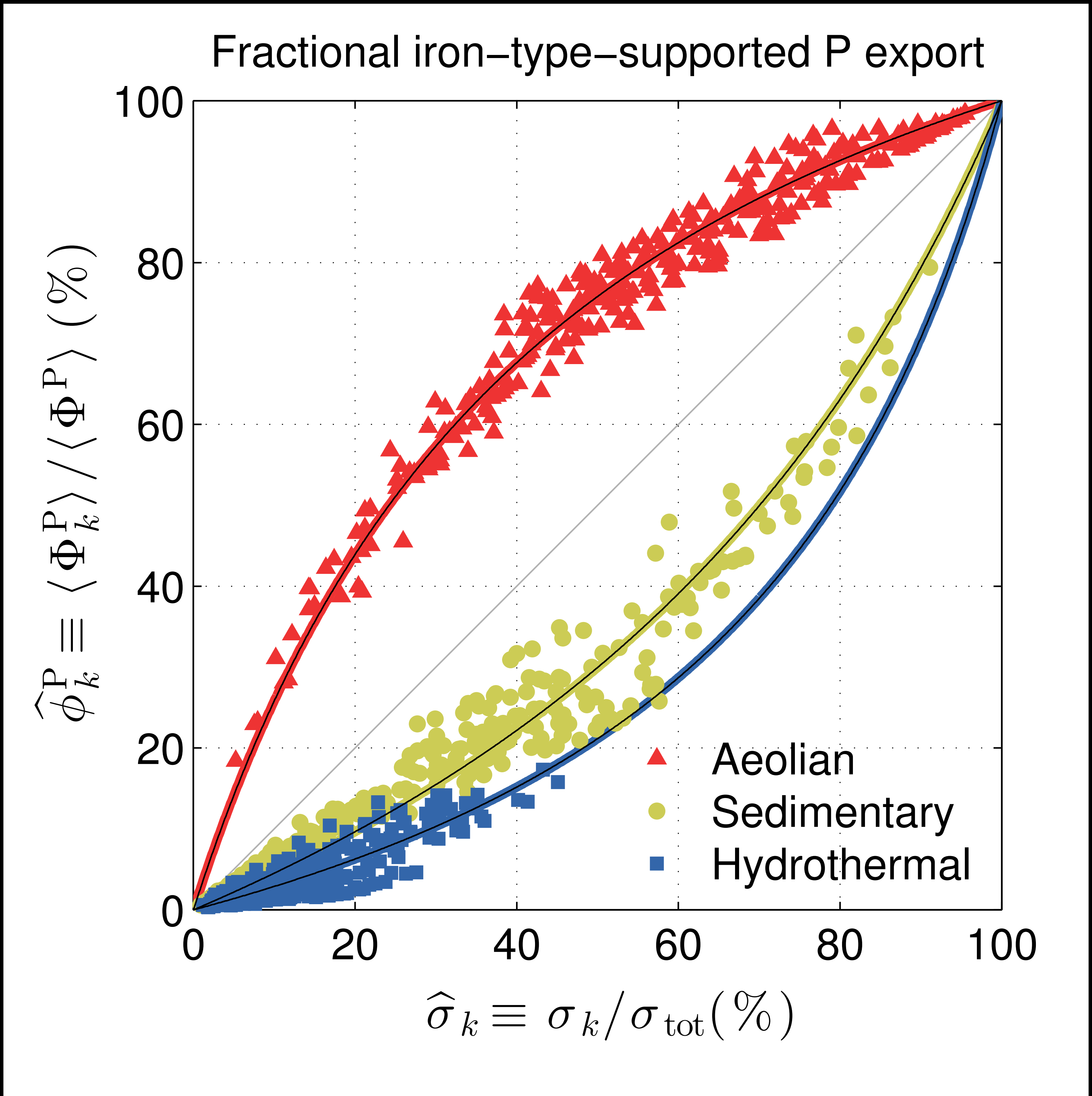
$$(\partial_t + L)x_{\text{Fe}} = \sum_k s_k$$

Allows non-invasive estimation of the true contribution of each source (s_k) to the total dFe:

$$x_k = L^{-1}s_k \qquad \text{with} \qquad x_{\text{Fe}} = \sum_k x_k$$

Apply to export production Φ:

$$\Phi_k = \Phi\frac{x_k}{x_{\text{Fe}}} \qquad \text{with} \qquad \Phi = \sum_k \Phi_k$$



Relative export-support efficiency:

$$e_A = 3.1 \pm 0.8$$

$$1/e_S = 2.3 \pm 0.6$$

$$1/e_H = 4. \pm 2.$$

Take home message

If you require long spin-ups, maybe you can solve for the steady state directly.

Provocative take home message

If you estimate contributions by computing anomalies and your system is nonlinear, maybe you are doing it wrong.