# An efficient inverse model of the ocean's coupled nutrient cycles.

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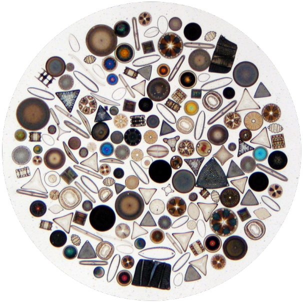


10 microns

# What is inverse modeling?

Model Parameters

- Uptake rate
- Export production
- Biogeochemical sources and sinks

Forward model →

Simulated Data

- Phosphate
- Silicic acid
- Dissolved iron

Model Parameters

- Uptake rate
- Export production
- Biogeochemical sources and sinks

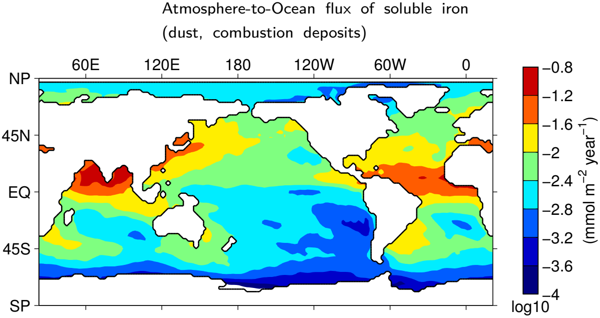
← Inverse model

Observed Data

- Phosphate
- Silicic acid
- Dissolved iron

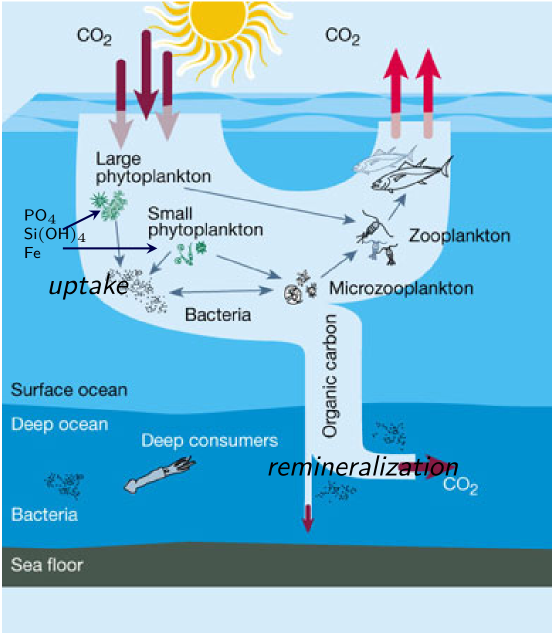
# Motivation

1. Understand and quantify how nutrient cycles are coupled
2. Quantify the pathways and timescales of the global response to large-scale iron perturbations
3. Quantify the role of marine diatoms and the silicon cycle in mediating the response
4. Quantify the sensitivity of elemental ratios (*e.g.*, Si : P) to iron pertrubations



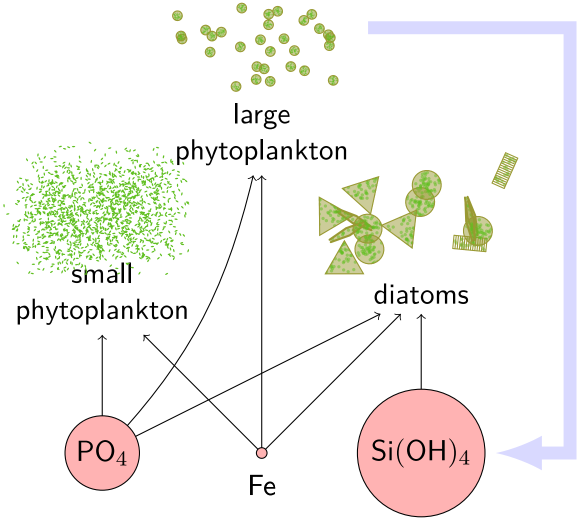

*Luo et al.* [2008]

# Nutrient cycle and coupling




- complex ecosystem
- large amount of species
- multiple nutrients
- poorly understood mechanisms (e.g. iron scavenging)
- simulation can be costly
- large parameter space
- lack of data (e.g. iron)
- poorly constrained parameters

# Distill essentials and simplify




- Zooplankton grazing embedded in phytoplankton mortality
- plankton not explicitly transported as tracers
- only nutrients tracers
- data constrained
- embedded in a data-assimilated global circulation
- numerically highly efficient; allows for optimization and novel diagnostics *Pasquier et al.* (in prep.)

# Plankton population model

- population local equation, a modified logistic equation, for each phytoplankton class

$$\partial_t\, b(t) = \mu\, b(t) - \lambda \left(\frac{b(t)}{b^*}\right)^{\frac{1}{\alpha}} b(t)$$

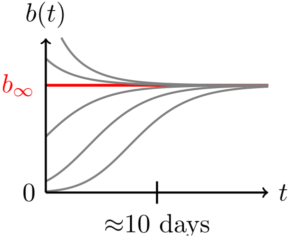
- assumed fast evolution to

$$b_\infty = b^* \left(\frac{\mu}{\lambda}\right)^{\alpha}$$

- associated nutrient uptake is population $\times$ growth rate

$$J = b_\infty\, \mu = b^* \left(\frac{\mu}{\lambda}\right)^{\alpha} \mu$$

• $b(t)$ plankton concentration
• $\mu$ growth rate
• $\lambda$ mortality rate
• $b^*$ pivotal concentration sets the scale of grazing




*Galbraith et al.* [2010]
*Dunne et al.* [2004]
*Armstrong* [1999, 2003]

# A versatile coupling approach

- mortality rate dependencies

$$\lambda = \lambda_0 \, F_T$$

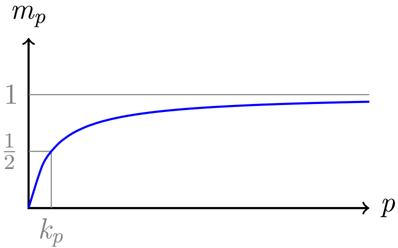
- growth rate dependencies

$$\mu = \mu_0 \, F_T \, F_I \min\left(m_P, \, r_{Fe} m_{Fe}, \, \dots\right)$$

- Michaelis-Menten kinetics

$$m_P = \frac{P}{P + k_P}$$

$$m_{Fe} = \frac{Fe}{Fe + k_{Fe}}$$




• $F_T = e^{\kappa T}$, temperature dependence *Eppley* [1972]
• $F_I$, irradiance dependence
• $m_X$, nutrient $X$ deficiency
• $P$, phosphate concentration
• $Fe$, iron concentration

*Galbraith et al.* [2010]

# Coupled nutrient cycling and biogenic transport embedded in global circulation

- total uptake is summed over phytoplankton classes $X$

$$J_{\mathsf{up}}(P, Fe, Si) = \sum_X J_X(P, Fe, Si)$$

- modeled $P$ and $Fe$ obey tracer equation

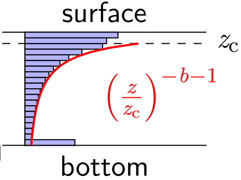
$$\partial_t P + \mathcal{T} P = \overbrace{-J_{\mathsf{up}}}^{\text{sink}} + \overbrace{\mathcal{S} J_{\mathsf{up}}}^{\text{source}}$$

$$\partial_t Fe + \mathcal{T} Fe = \overbrace{R_{Fe:P}(Fe)}^{\text{uptake ratio}}(-J_{\mathsf{up}} + \mathcal{S} J_{\mathsf{up}}) + \overbrace{J_{\mathsf{ext}(Fe)}}^{\text{external sources and sinks}}$$

- similar equation for Si

• $\mathcal{T}$ data-assimilated transport operator *Primeau et al.* [2013]

• $\mathcal{S}$ (biogenic transport) instantly redistributing $J_{\mathsf{up}}$ as the sinking particle flux divergence




*Martin et al.* [1990]

# Discretized equations

An expression of the form $\partial_t \mathbf{x} = \mathbf{f}(\mathbf{x})$ where the variable $\mathbf{x}$ is

$$\mathbf{x} = \begin{bmatrix} \mathbf{p} \\ \mathbf{f} \\ \mathbf{s} \end{bmatrix} \qquad \text{with } \mathbf{p} = \begin{bmatrix} P_1 \\ P_2 \\ \vdots \\ P_n \end{bmatrix}, \mathbf{f} = \begin{bmatrix} Fe_1 \\ Fe_2 \\ \vdots \\ Fe_n \end{bmatrix}, \text{and } \mathbf{s} = \begin{bmatrix} Si_1 \\ Si_2 \\ \vdots \\ Si_n \end{bmatrix}.$$

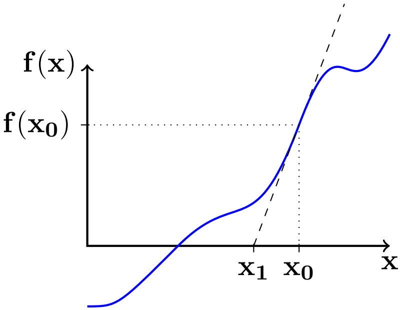
and the right-hand side function is

$$\mathbf{f}(\mathbf{x}) = \overbrace{-\begin{bmatrix} \mathbf{T} & & \\ & \mathbf{T} & \\ & & \mathbf{T} \end{bmatrix} \mathbf{x}}^{\text{linear}} + \overbrace{\begin{bmatrix} (\mathbf{S}-\mathbf{I})\mathbf{J}_{\mathsf{up}}(\mathbf{x}) \\ (\mathbf{S}-\mathbf{I})\mathrm{diag}(\mathbf{R}_{\mathbf{f}:\mathbf{p}})\mathbf{J}_{\mathsf{up}}(\mathbf{x}) + \mathbf{J}_{\mathsf{ext}(\mathbf{Fe})} \\ (\mathbf{S}-\mathbf{I})\mathrm{diag}(\mathbf{R}_{\mathbf{s}:\mathbf{p}})\mathbf{J}_{\mathsf{up}}(\mathbf{x}) + \mathbf{J}_{\mathsf{ext}(\mathbf{Si})} \end{bmatrix}}^{\text{nonlinear}}$$

# Newton PDE solution

- steady state: $\partial_t \mathbf{x} = \mathbf{f}(\mathbf{x}) = \mathbf{0}$
- use Newton's Method (generalized zero search)
  linear approximation:

$$\mathbf{f}(\mathbf{x_1}) = \mathbf{f}(\mathbf{x_0}) + \mathbf{Df}(\mathbf{x_0})(\mathbf{x_1} - \mathbf{x_0}) + \mathbf{o}(\|\mathbf{x_1} - \mathbf{x_0}\|)$$

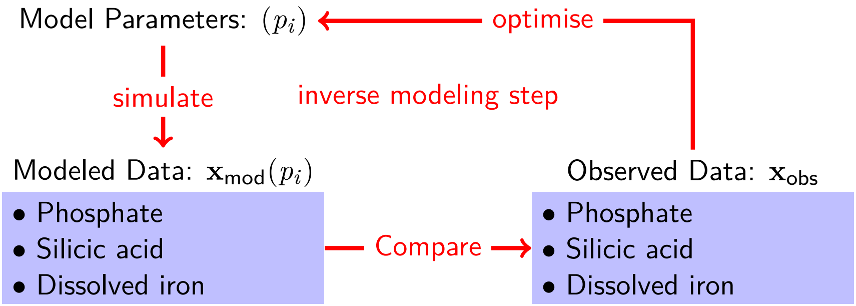



where $\mathbf{Df}$ is the Jacobian,
a $n \times n$ sparse matrix
where $n \approx \mathbf{200000}k$
for $k$ nutrients!

To get $\mathbf{f}(\mathbf{x_1}) \approx \mathbf{0}$,
we take

$$\mathbf{x_1} = \mathbf{x_0} - \mathbf{Df}(\mathbf{x_0})^{-1}\mathbf{f}(\mathbf{x_0})$$

# Objective parameter determination



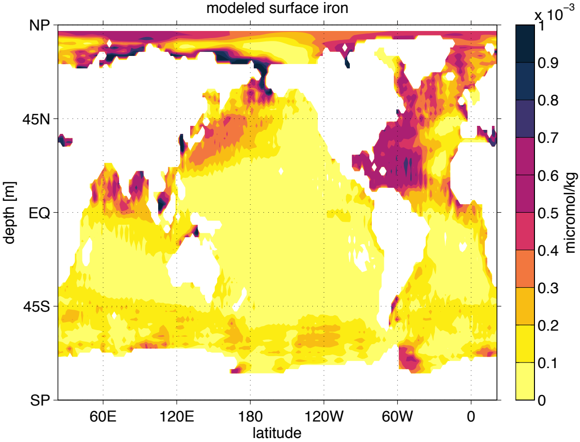

- Chosing appropriate weights $\mathbf{w}$, we build an objective function of the quadratic concentration mismatch:

$$c(p_i) = (\mathbf{x}_{\mathsf{mod}} - \mathbf{x}_{\mathsf{obs}})^T \operatorname{diag}(\mathbf{w})(\mathbf{x}_{\mathsf{mod}} - \mathbf{x}_{\mathsf{obs}})$$

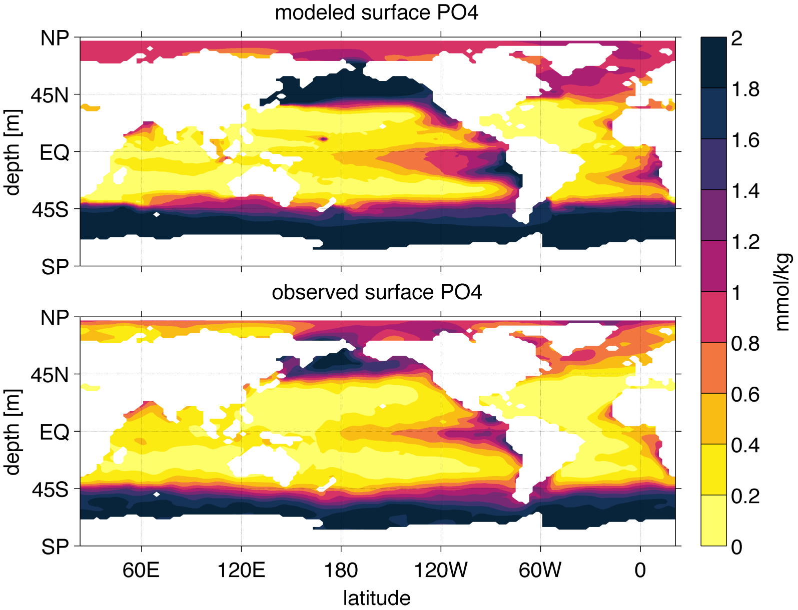
- nonlinear w.r.t. parameters
- optimisation only possible with efficient simulation

# Preliminary model setup

- prescribed iron from *Frantz et al.* (in prep.)
- some data interpolated from different grid
- only 2 parameters optimised



# Model vs. observed Phosphate



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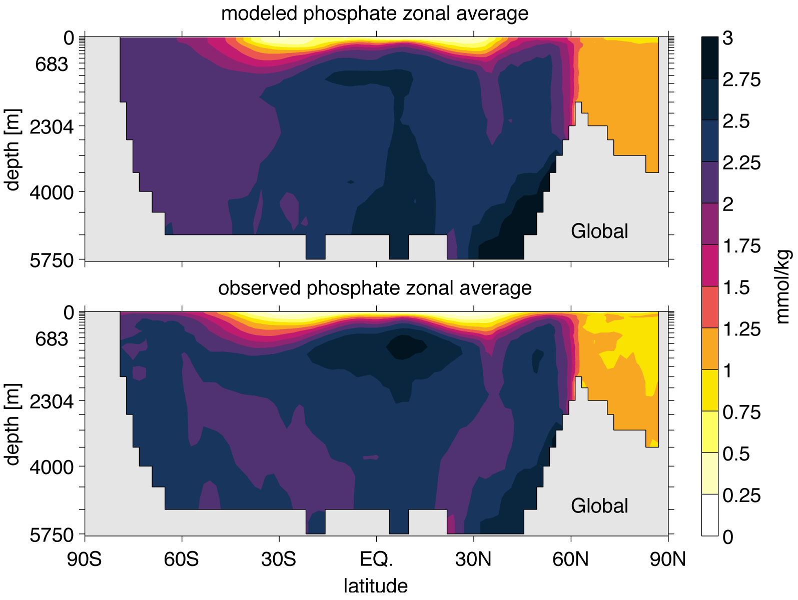

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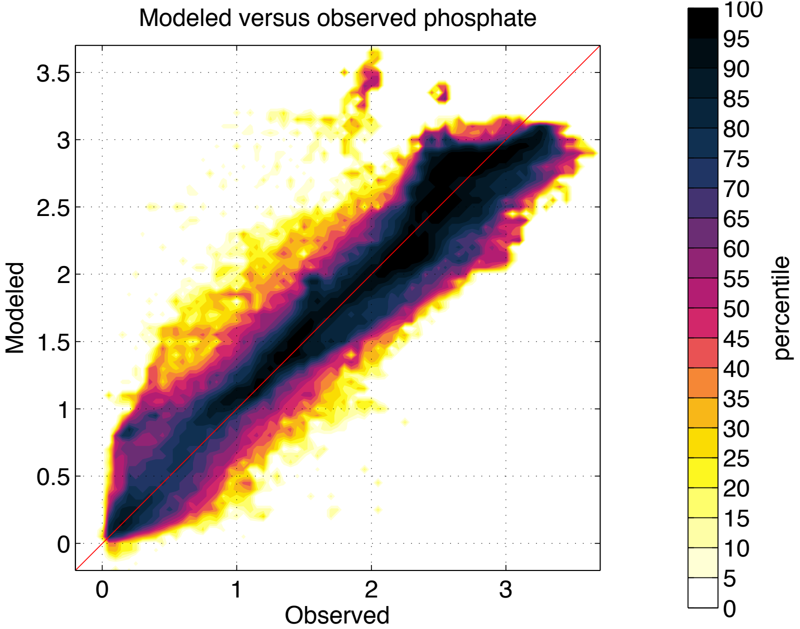
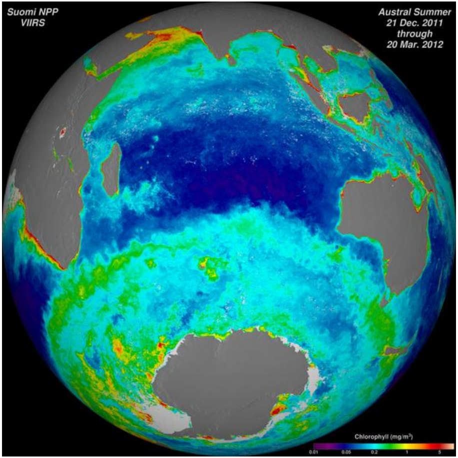


# Future outlook and conclusion

- from prescribed iron to coupled iron
- refine coupling parameters
- optimise larger set of parameters
- formulate iron perturbations
- diagnose the teleconnections of the response using Green functions forward and adjoint models *Pasquier et al.* (in prep.)
- include silicon cycle
- elucidate elemental ratios



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