Exploring iron control on global productivity: "FePSi", an inverse model of the ocean's coupled phosphorus, silicon and iron cycles.

Benoît Pasquier - Ph.D. student

FePSi

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1. Can we constrain the BGC parameters of the nutrient cycles?
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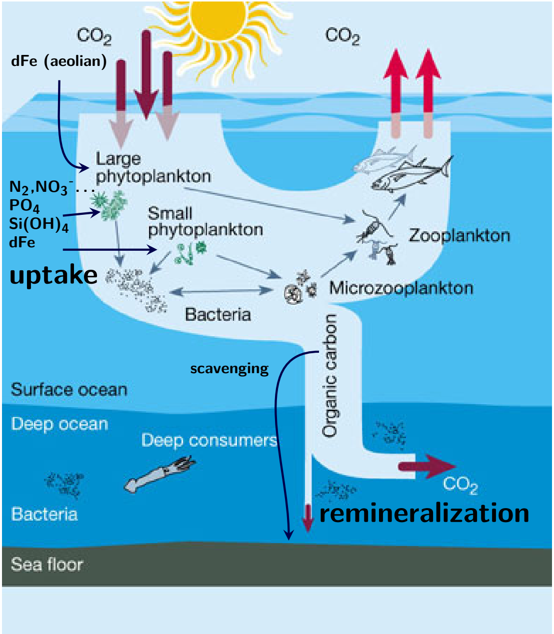
3. Can we test the $Si(OH)_4$ leakage hypothesis?

Nutrient cycle and coupling



In the real world:

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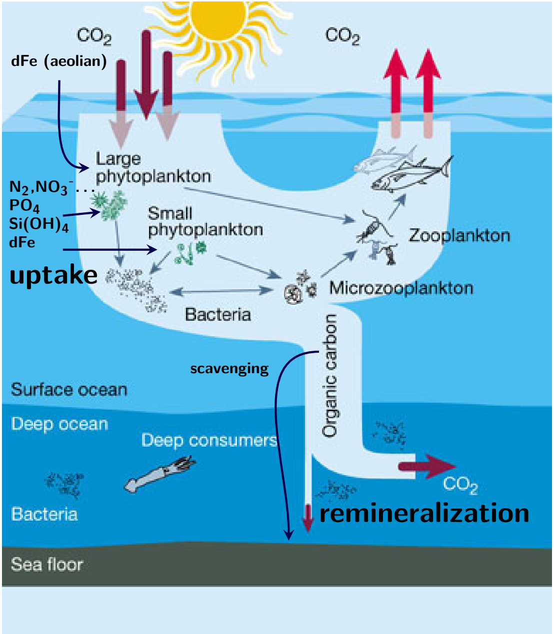

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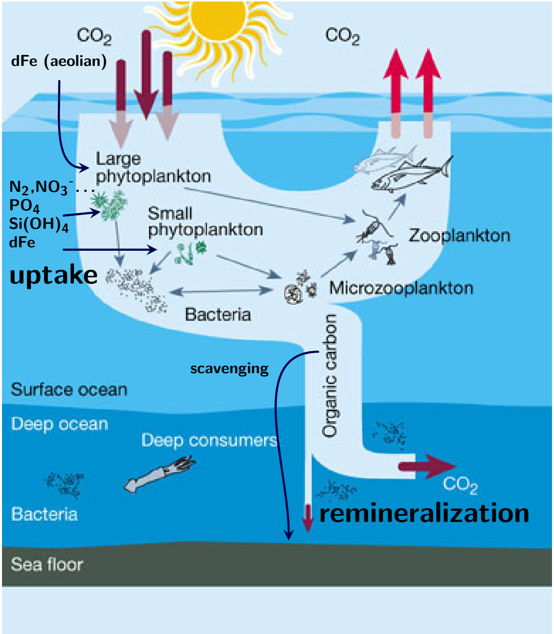

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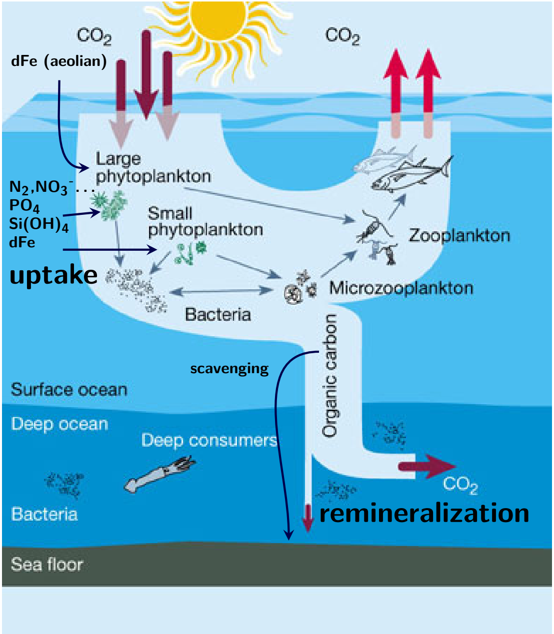

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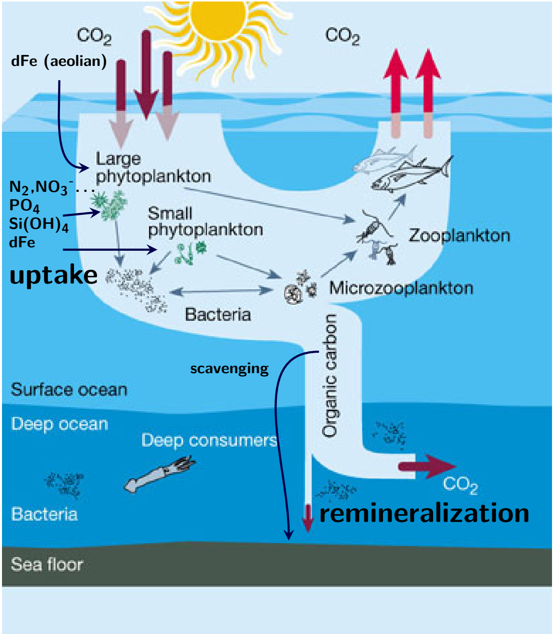

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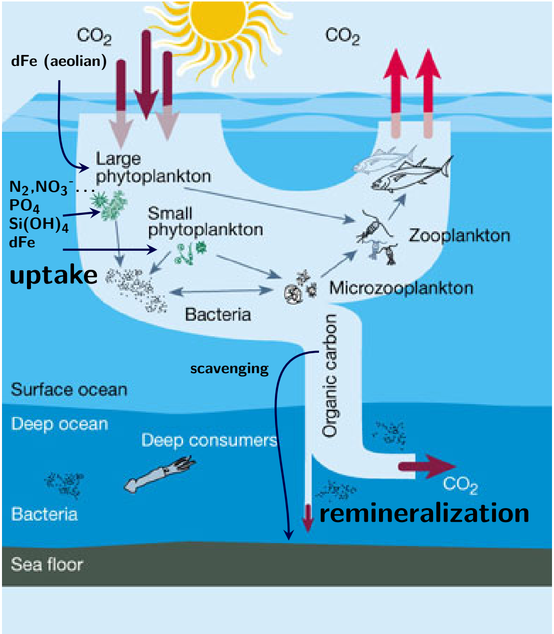

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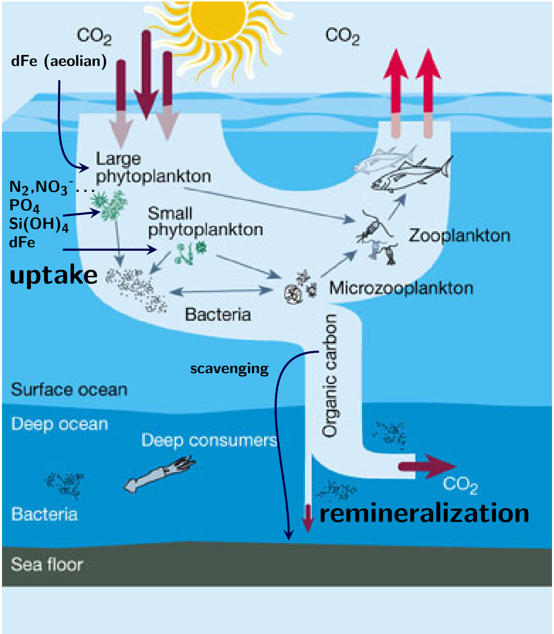

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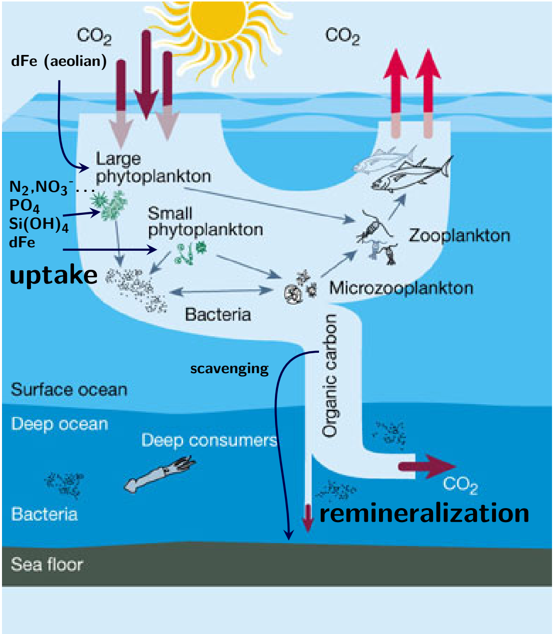

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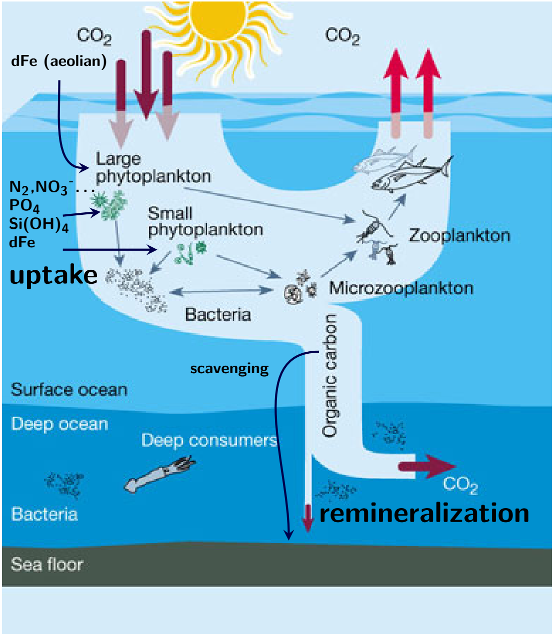

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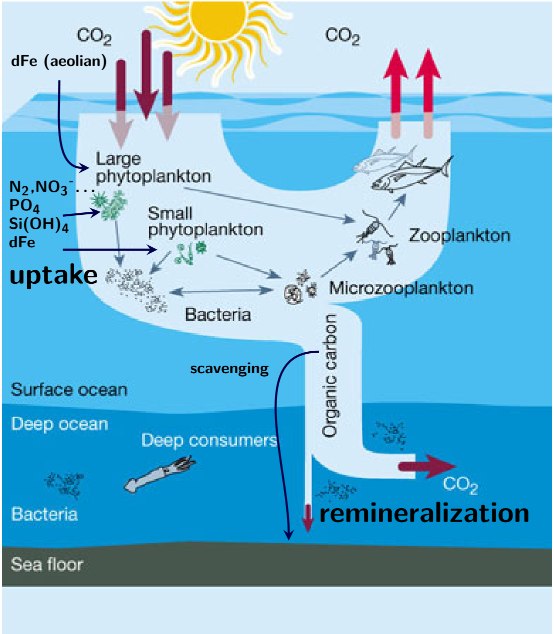

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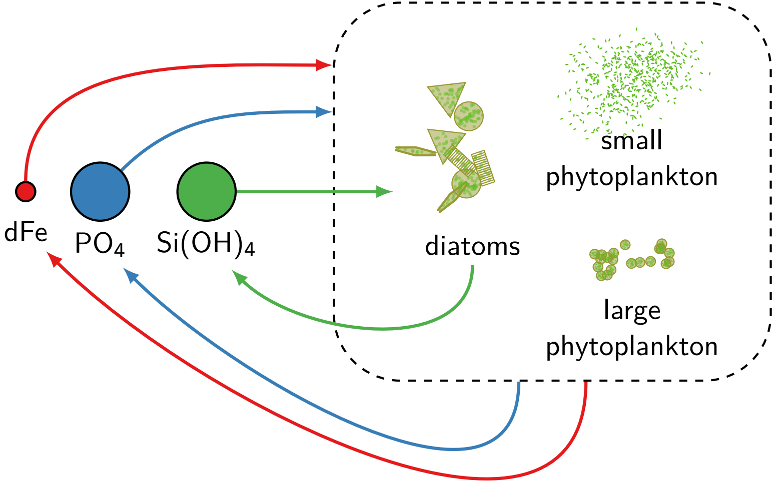
- poor constraints on parameters
- lack of data (iron)

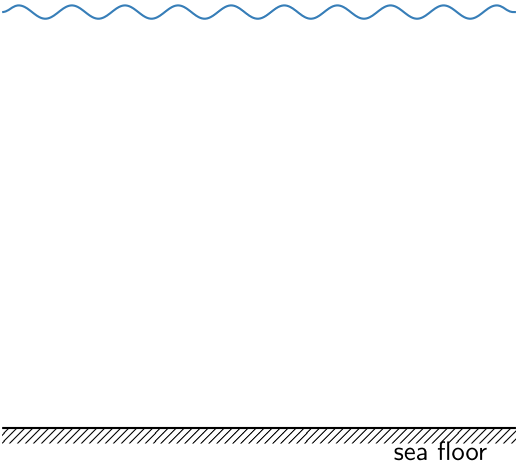
Simplified biological cycling, b



- 3 nutrients
- 3 phytoplankton classes (not transported)
- No zooplankton

Galbraith et al., 2010; *Matsumoto et al.*, 2013

Simplified iron external source and sinks



Parekh et al., 2004;
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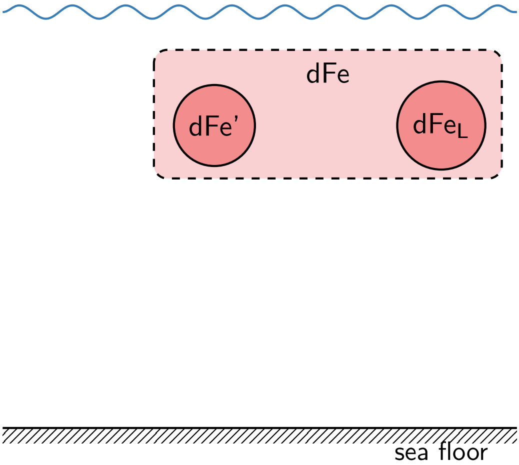

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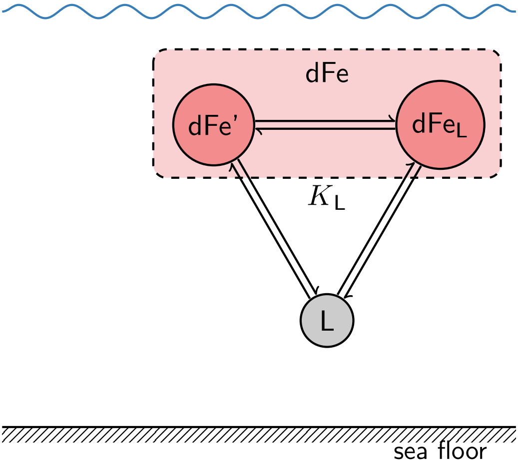

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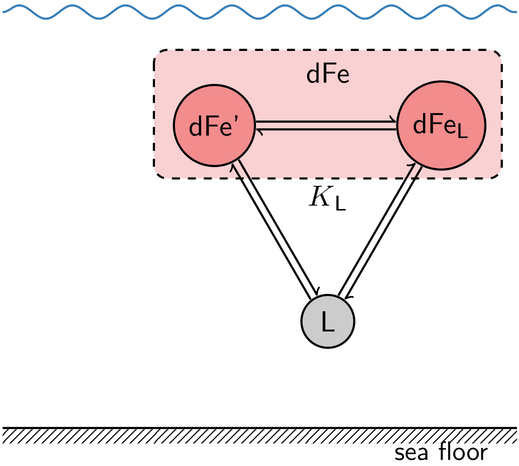

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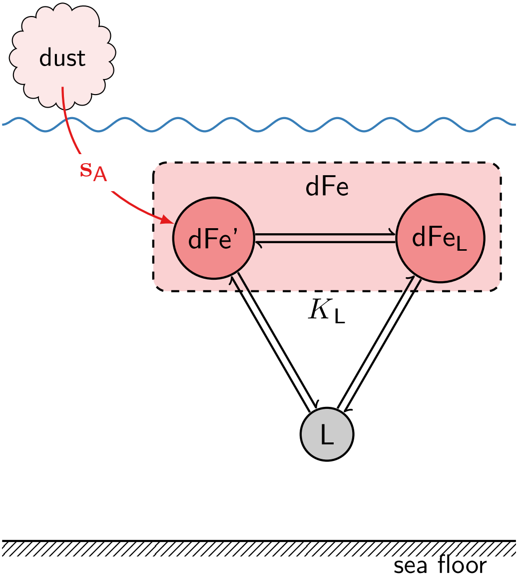

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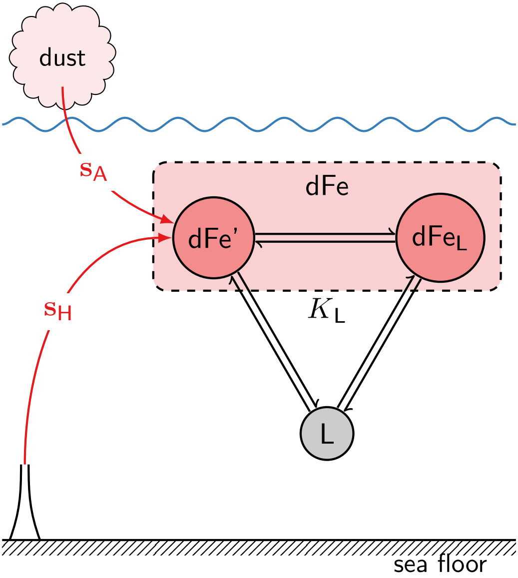

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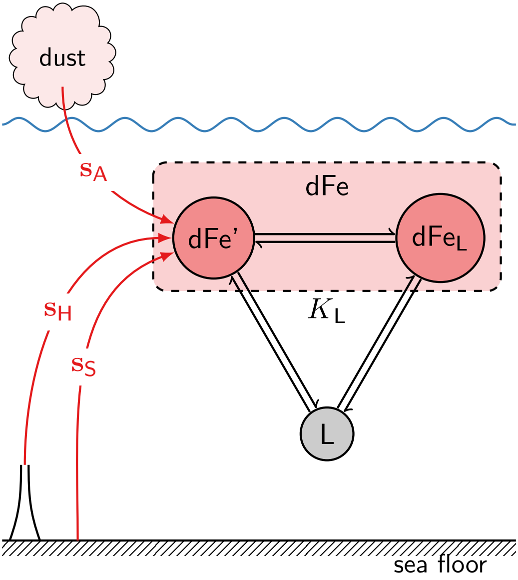

- $dFe = dFe' + dFe_L$
- Chemical equilibrium with L (constant K_L)
- 3 sources of dFe':
  - Aeolian
  - Hydrothermal
  - Sedimentary

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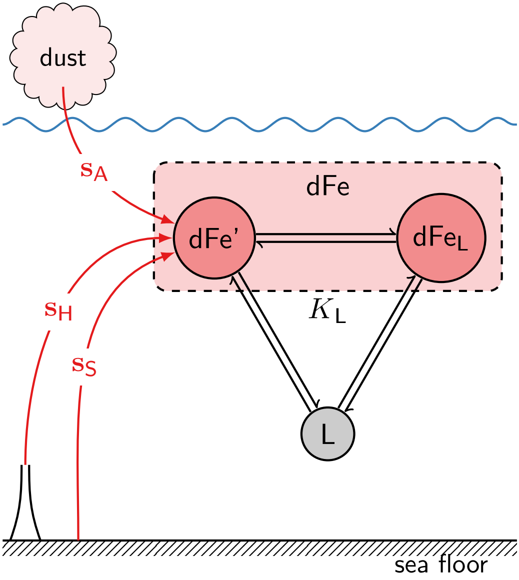

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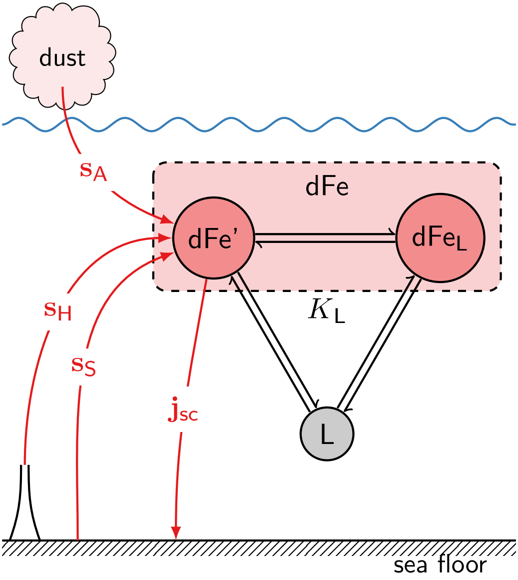

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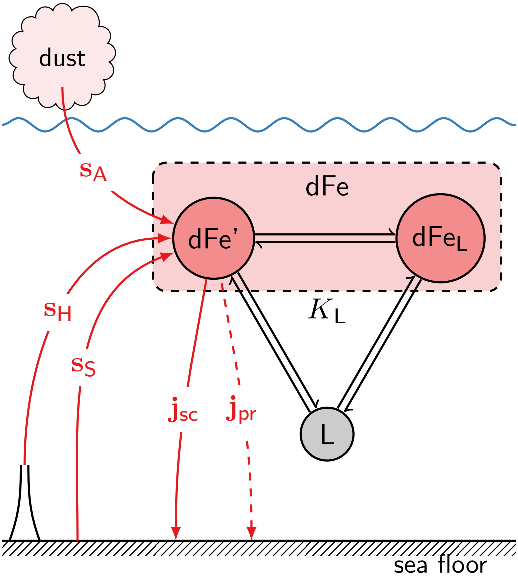

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- 3 sources of dFe':
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  - Hydrothermal
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- 2 sinks of dFe':
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  - Precipitation

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Discretized PDE

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- We solve the steady state equation $\mathbf{f}(\mathbf{x}) = \mathbf{0}$ using Newton's Method

Objective parameter determination

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Model Parameters: (p_i)

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↓

simulate

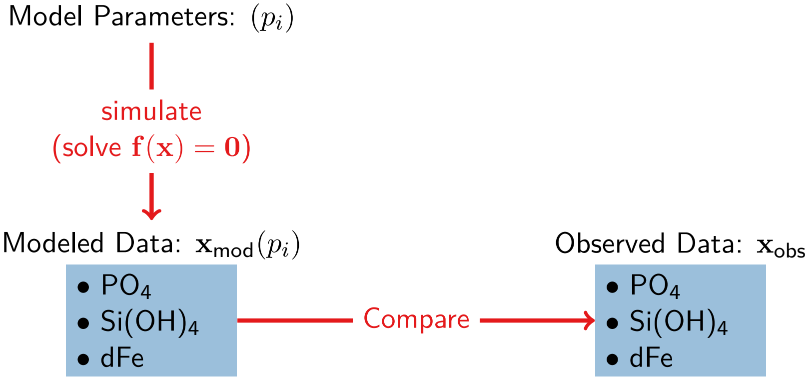
(solve $\mathbf{f}(\mathbf{x}) = \mathbf{0}$)

↓

Modeled Data: $\mathbf{x}_{\mathsf{mod}}(p_i)$

- PO_4
- $Si(OH)_4$
- dFe

Objective parameter determination



- Chosing appropriate weights $\mathbf{w}$, we build an objective function of the quadratic concentration mismatch:

$$c(p_i) = (\mathbf{x}_{\text{mod}} - \mathbf{x}_{\text{obs}})^{\mathsf{T}} \text{diag}(\mathbf{w})(\mathbf{x}_{\text{mod}} - \mathbf{x}_{\text{obs}})$$

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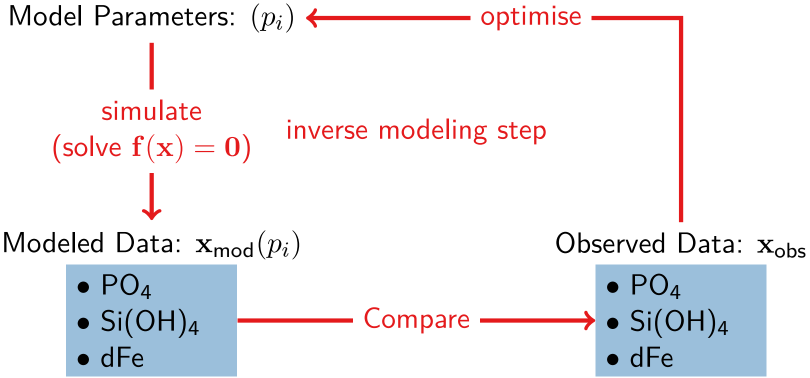

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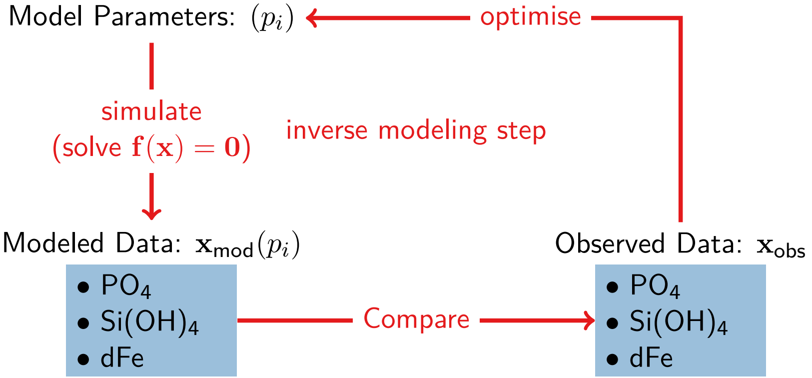

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- optimisation only possible with efficient simulation

FePSi key parameters

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- Biological parameters (optimized):
  - Uptake rate timescale τ
  - Phytoplankton populations P^*_{dia}, P^*_{lrg}, and P^*_{sml}
  - (Fe:P) uptake ratio: $R_{\mathsf{f:p}}$ and $k_{\mathsf{f:p}}$
  - (Si:P) uptake ratio: $(\mathsf{Si}:\mathsf{P})_{\mathsf{min}}$, k_1, k_2, and k_3

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- Iron sources and sinks (optimized for family of σ_{A}):
  - Sedimentary source strength σ_{S}
  - Hydrothermal source strengths $\sigma_{\mathsf{H}}^{\mathsf{PAC}}$,$\sigma_{\mathsf{H}}^{\mathsf{ATL}}$,$\sigma_{\mathsf{H}}^{\mathsf{IND}}$, and $\sigma_{\mathsf{H}}^{\mathsf{SO}}$
  - Organic scavenging strength κ^{org} and profile shape β
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  - Organic scavenging strength κ^{org} and profile shape β
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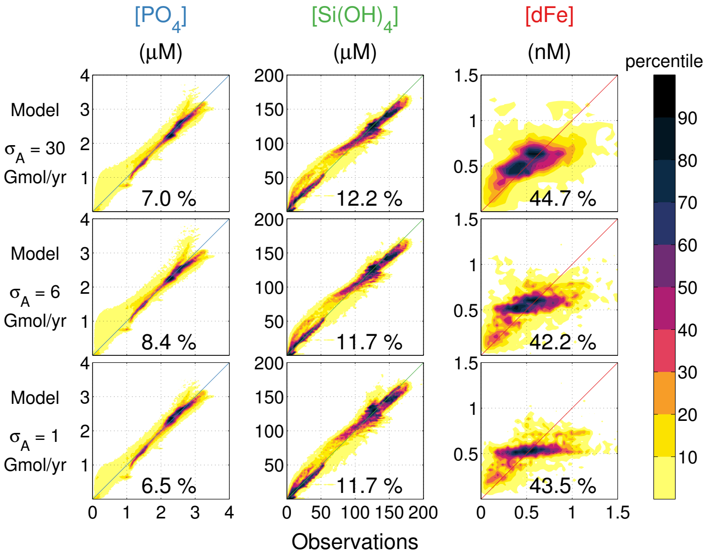
- Non optimized (yet) parameters:
  - Half-saturation rates: $k_{\mathsf{dia}}^{\mathsf{P}}$, $k_{\mathsf{dia}}^{\mathsf{Fe}}$, $k_{\mathsf{dia}}^{\mathsf{Si}}$, $k_{\mathsf{lrg}}^{\mathsf{P}}$, $k_{\mathsf{lrg}}^{\mathsf{Fe}}$, $k_{\mathsf{sml}}^{\mathsf{P}}$, and $k_{\mathsf{sml}}^{\mathsf{Fe}}$
  - local recycling fractions: σ_{dia}, σ_{lrg}, and σ_{sml}
  - sinking particle profiles: b_{dia}, b_{lrg}, and b_{sml},
  - light harvesting efficiency: $\alpha_{\mathsf{min}}^{\mathsf{chl}}$, $\alpha_{\mathsf{max}}^{\mathsf{chl}}$, $\theta_{\mathsf{min}}^{\mathsf{chl}}$, and $\theta_{\mathsf{max}}^{\mathsf{chl}}$
  - ligand stability constant K_{L}

Climatological base state - mismatch joint PDF



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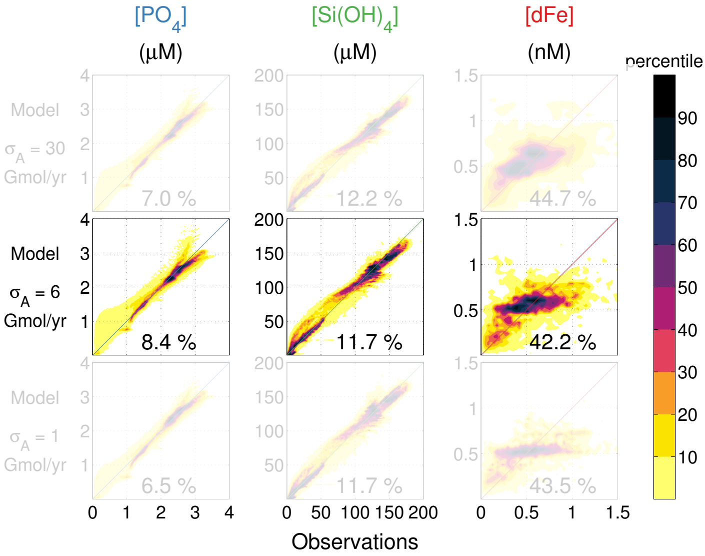
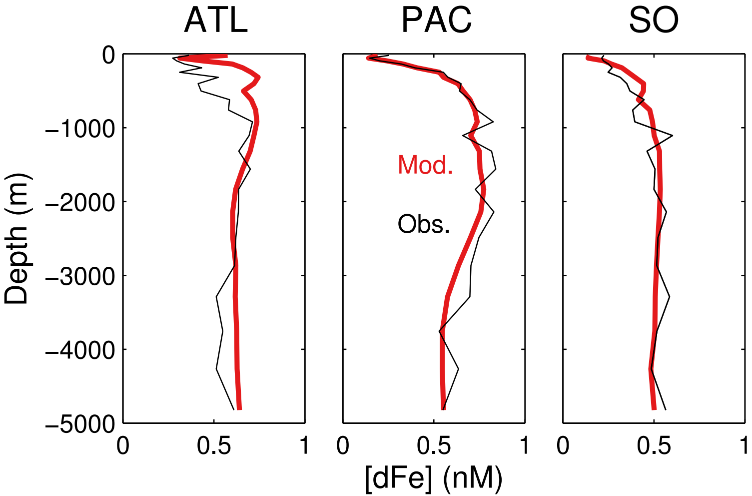


Climatological base state - iron profiles

Model with $\sigma_{\mathrm{A}} = 6\,\mathrm{Gmol_{dFe}/\,yr}$



Climatological base state - limiting nutrients

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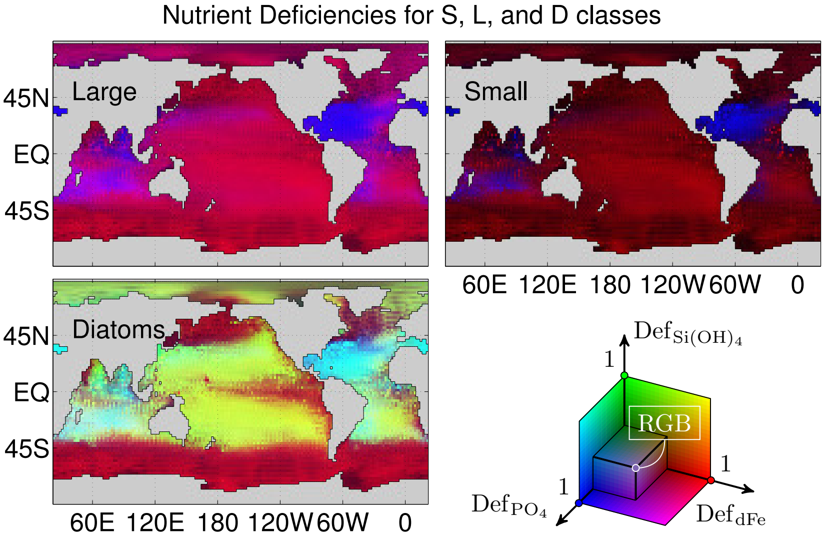

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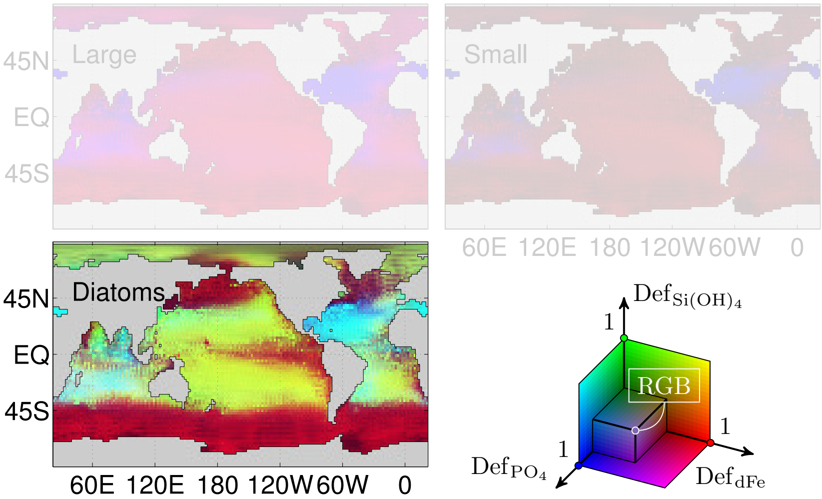
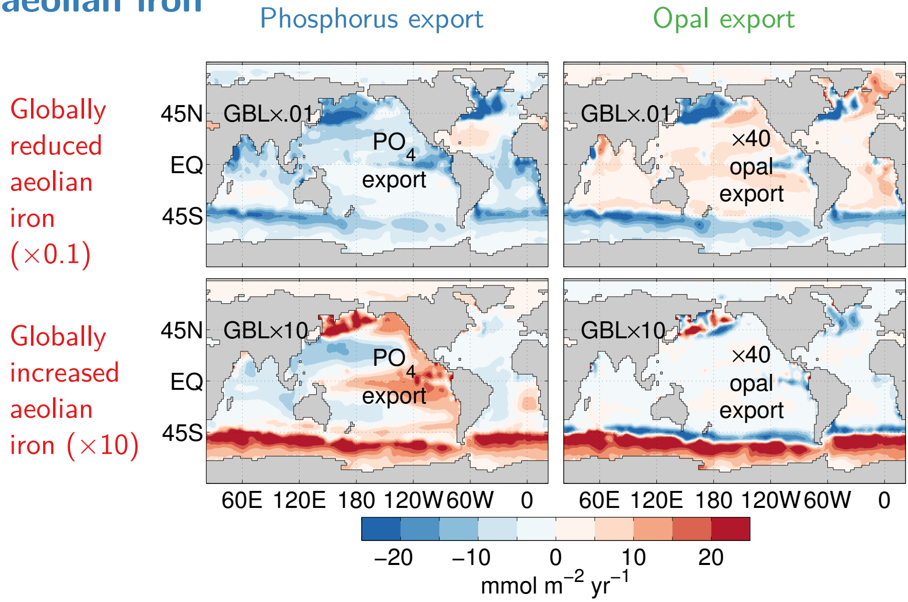
Nutrient Deficiencies for S, L, and D classes



Export production anomaly due to perturbed aeolian iron

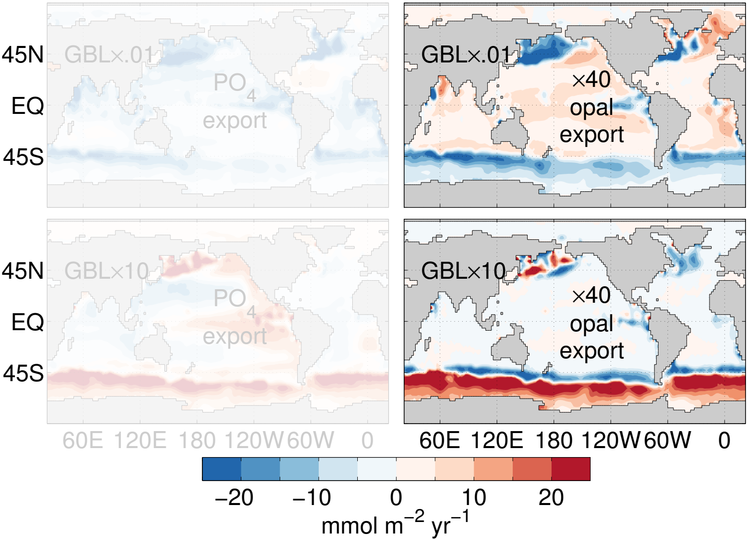


Export production anomaly due to perturbed aeolian iron

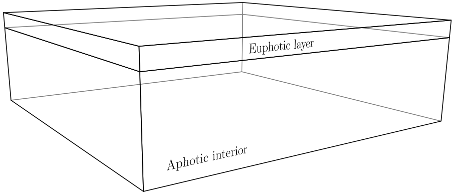
Phosphorus export | Opal export

Globally reduced aeolian iron (×0.1)

Globally increased aeolian iron (×10)



Path densities of regenerated nutrients



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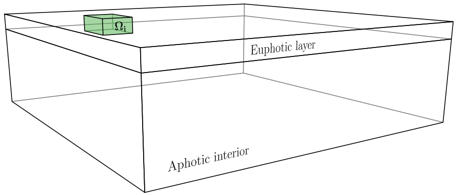

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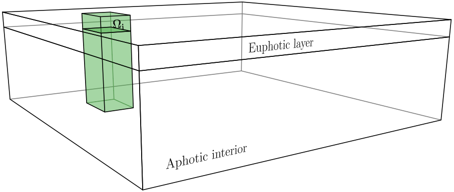

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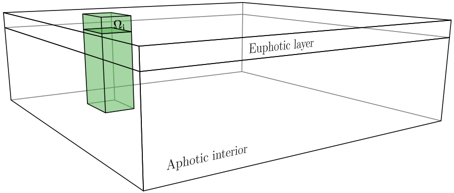

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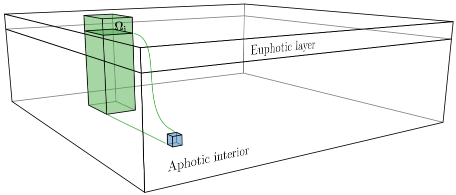

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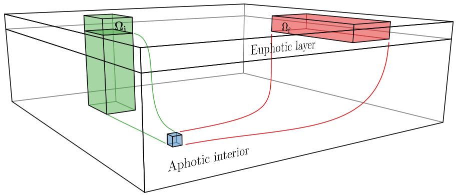

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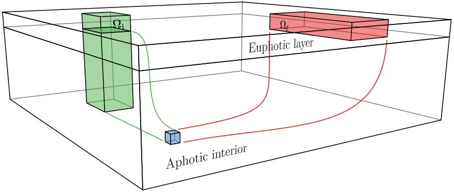

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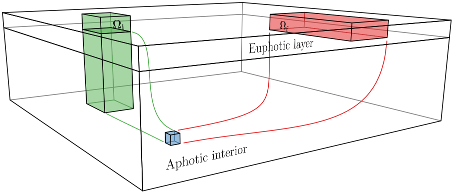
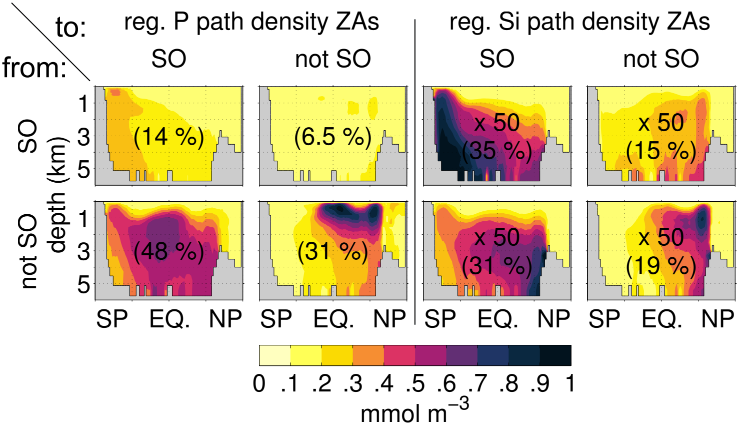


6. Combine into path density:

$$\langle \boldsymbol{\eta}_{\mathsf{reg}} \rangle = \langle \tilde{\boldsymbol{\mathcal{G}}}_{\mathsf{reg}} \rangle \odot \langle \mathbf{g}_{\mathsf{reg}} \rangle$$ (element-wise multiplication)

PD of regenerated PO_4 and $Si(OH)_4$ - base state



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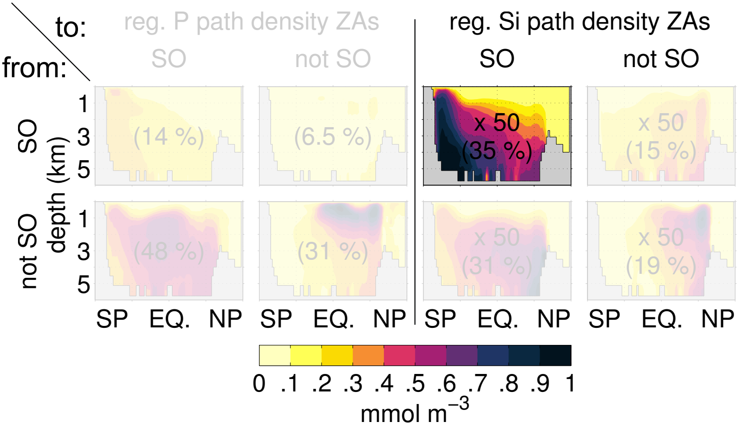

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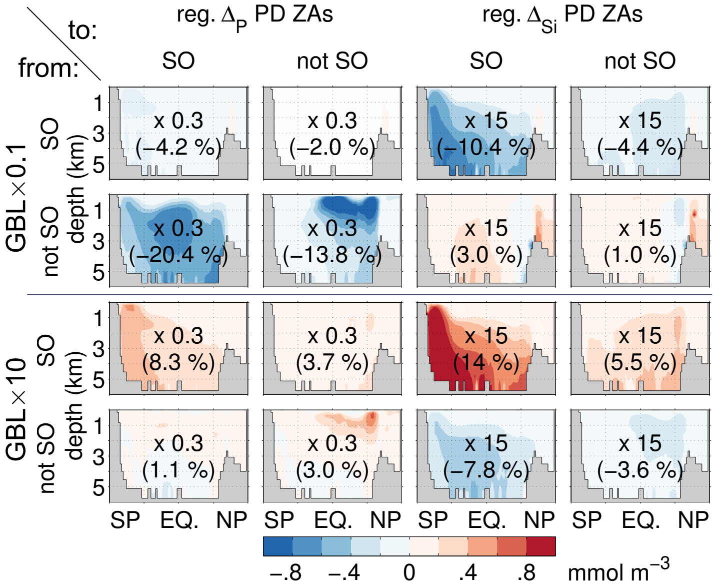

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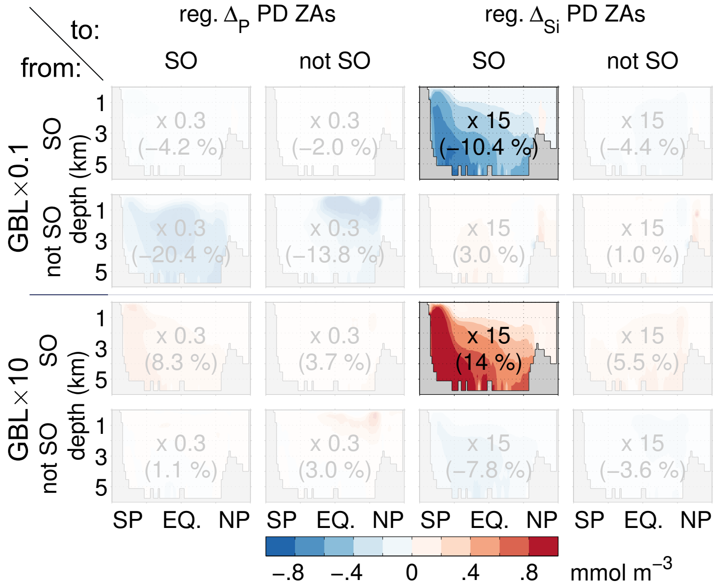

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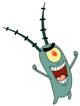
Questions?

FePSi



Newton PDE solution

- steady state: $\partial_t \mathbf{x} = \mathbf{f}(\mathbf{x}) = \mathbf{0}$
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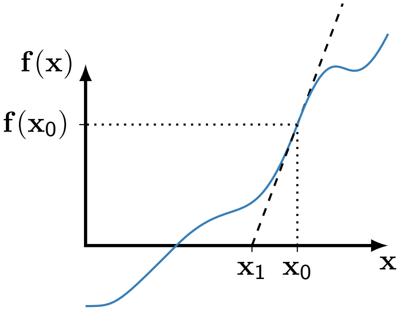
$$\mathbf{f}(\mathbf{x}_1) = \mathbf{f}(\mathbf{x}_0) + \mathbf{Df}(\mathbf{x}_0)(\mathbf{x}_1 - \mathbf{x}_0) + o(\|\mathbf{x}_1 - \mathbf{x}_0\|)$$



where $\mathbf{Df}$ is the Jacobian,
a $n \times n$ sparse matrix
where $n \sim$ 600,000!

To get $\mathbf{f}(\mathbf{x}_1) \sim \mathbf{0}$,
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Kelley, 2003

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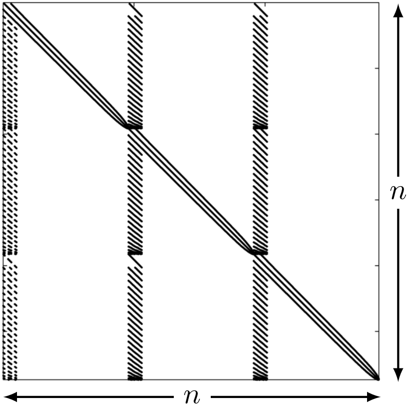

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